Assignment for Tuesday, 1/16

I. Exercises from book:
   Section 1.4: #1c, #3, #10, #14

II. Supplementary Exercises:
   1. Use Well-Ordering Principle to show that \( \sqrt{2} \) is irrational.
   2. Prove that the expression \( 3^{3n+3} - 26n - 27 \) is a multiple of 169 for all \( n \geq 1 \).
   3. If \( x \) is any real number other than 1, then
      \[
      \sum_{j=0}^{n-1} x^j = \frac{x^n - 1}{x - 1}
      \]

III. Optional Exercises: (Only if you have finished Ⅰ&Ⅱ)
   1. An integer is called good if we can write \( n = a_1 + a_2 + ... + a_k \), where
      \( a_1, a_2, ..., a_k \) are positive integers (not necessarily distinct) satisfying
      \[
      \frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_k} = 1
      \]
      Given that the integers 33, 34, ..., 73 are good, prove that every integer \( \geq 33 \) is good.
   2. Prove the AM-GM inequality using induction.
      AM-GM inequality: Let \( a_1, ..., a_n \) be non-negative real numbers, then
      \[
      (a_1, ..., a_n)^{1/n} \leq \frac{1}{n} \sum_{i=1}^{n} a_i
      \]