# Assignment for Tuesday, 1/16

I) Exercises from book:
Section 1.1 → #1ce, #3, #10, #14

II) Supplementary Exercises:

1) Use Well-Ordering Principle to show that $\sqrt{2}$ is irrational

2) Prove that the expression $3^{3n+3} - 26n - 27$ is a multiple of 169 for all $n \in \mathbb{N}$.

3) If $x$ is any real number other than 1, then

$$\sum_{j=0}^{n-1} x^j = \frac{x^n - 1}{x - 1}$$

III) Optional ~~Challenging~~ Exercises: (Only if you have finished I & II)

1) An integer is called good if we can write $n = a_1 + a_2 + \cdots + a_k$, where $a_1, a_2, \ldots, a_k$ are positive integers (not neccessarily distinct) satisfying

$$\frac{1}{a_1} + \cdots + \frac{1}{a_k} = 1$$

Given that the integers 33, 34, ..., 73 are good Prove that every integer $\geq 33$ is good.

2) Prove the AM-GM inequality using induction.
AM-GM ineq.: Let $a_1, \ldots, a_n$ be non-neg. real numbers, then $(a_1 \cdots a_n)^{1/n} \leq \frac{1}{n} \sum_{i=1}^{n} a_i$