Assignment for Thursday 2/1

(I) Exercises from the book:

Section 3.1 → 3cd, 4, 6bc, 9, 11, 16, 17

(II) Supplementary Exercises:

(11) Prove that if $2^n - 1$ is prime then n is prime.

(Hint: Prove the contrapositive.)
[Compare this to 11b above]

(12) Let $F_n = 2^{2^n} + 1$, $n \geq 0$ (These are called Fermat Numbers)

Show that $\gcd(F_m, F_n) = 1$, for $m > n \geq 0$.

(III) Optional Exercises:

(4) Prove that in any set of 33 distinct integers with prime factors amongst $\{5, 7, 11, 13, 23\}$, there must be two whose product is a square.

(5) Prove that there is exactly one natural number n for which $2^8 + 2^{11} + 2^n$ is a perfect square.