Assignment for Thursday 2/1

I. Exercises from the book:

Section 3.1 → 3cd, 4, 6bc, 9, 11, 16, 17

II. Supplementary Exercises:

11. Prove that if \( 2^n - 1 \) is prime then \( n \) is prime.
   (Hint: Prove the contrapositive.)
   [Compare this to 11b above]

12. Let \( F_n = 2^{2^n} + 1 \), \( n \geq 0 \) (These are called Fermat Numbers).
    Show that \( \gcd(F_m, F_n) = 1 \), for \( m > n \geq 0 \).

III. Optional Exercises:

4. Prove that in any set of 33 distinct integers with prime factors amongst \( 5, 7, 11, 13, 23 \),
   there must be two whose product is a square.

5. Prove that there is exactly one natural number \( n \) for which \( 2^8 + 2^n + 2^m \) is a perfect square.