Another Proof of "\(\sqrt{2}\) is irrational"

Fermat's method of infinite descent:

This is disguised form of getting a contradiction to the Well Ordering Principle by producing an infinite sequence of decreasing positive integers if \(\sqrt{2}\) is assumed to be rational.

Suppose \(\sqrt{2} = \frac{a}{b}\)

Let \(a_1 = 2b - a\) & \(b_1 = a - b\)

then \(\frac{a}{b} = \frac{a_1}{b_1}\)  \(\therefore a^2 = 2b^2\)

Also, \(0 < b_1 < b\)

since \(1 < \frac{a}{b} < 2 \Rightarrow b < a < 2b \Rightarrow 0 < a - b < b\)

Now, define \(a_{k+1} = 2b_k - a_k\) & \(b_{k+1} = a_k - b_k\), \(k \geq 1\)

& by same reasoning we get \(\sqrt{2} = \frac{a}{b} = \frac{a_{k+1}}{b_{k+1}}\)

and \(0 < b_{k+1} < b_k < b\), \(k \geq 1\)

giving an infinite sequence of positive integers \(b_1, b_2, \ldots\)

which is not possible.

Question: Use this idea to prove that \(\sqrt{N}\) is irrational where \(N\) is not a square

(How will you define \(a_1, b_1\) in this case?)