

Some Examples (Section 4.4)

① Solve the system of congruences

$$17x \equiv 3 \pmod{2}$$

$$17x \equiv 3 \pmod{3}$$

$$17x \equiv 3 \pmod{5}$$

$$17x \equiv 3 \pmod{7}$$

Soln.

Simplify:

$$17x \equiv 3 \pmod{2} \Leftrightarrow 17x = 2k + 3 \Leftrightarrow x = 2(k - 8x + 1) + 1 \Leftrightarrow x \equiv 1 \pmod{2}$$

$$17x \equiv 3 \pmod{3} \Leftrightarrow 17x = 3l + 3 \Leftrightarrow 2x = 3(l - 5x + 1) + 0 \Leftrightarrow 2x \equiv 0 \pmod{3}$$

$$17x \equiv 3 \pmod{5} \Leftrightarrow 17x = 5m + 3 \Leftrightarrow 2x = 5(m - 3x) + 3 \Leftrightarrow 2x \equiv 3 \pmod{5}$$

$$17x \equiv 3 \pmod{7} \Leftrightarrow 17x = 7n + 3 \Leftrightarrow 3x = 7(n - 2x) + 3 \Leftrightarrow 3x \equiv 3 \pmod{7}$$

Also, note that $2x \equiv 0 \pmod{3} \Leftrightarrow x \equiv 0 \pmod{3}$

since $\gcd(2, 3) = 1$ we can cancel 2.

and $3x \equiv 3 \pmod{7} \Leftrightarrow x \equiv 1 \pmod{7}$

since $\gcd(3, 7) = 1$ we can cancel 3.

Finally, $x \equiv 4 \pmod{5} \Leftrightarrow x = 5k + 4 \Rightarrow 2x = 5k' + 8$

$$\Rightarrow 2x = 5k'' + 3$$

$$\Leftrightarrow 2x \equiv 3 \pmod{5}$$

Thus, it is enough to solve the system

$$x \equiv 1 \pmod{2}$$

$$x \equiv 0 \pmod{3}$$

$$x \equiv 4 \pmod{5}$$

$$x \equiv 1 \pmod{7}$$

which can be done either using CRT or by direct substitutions.

② The system $a \equiv 0 \pmod{2}$

$$a \equiv 2 \pmod{3}$$

$$a \equiv 2 \pmod{6}$$

is equivalent to $a \equiv 0 \pmod{2}$
the smaller system $a \equiv 2 \pmod{3}$

because:

$$a \equiv 0 \pmod{2} \Rightarrow a = 2l$$

$$a \equiv 2 \pmod{3} \Rightarrow a = 3k + 2$$

$$\Rightarrow 2l = 3k + 2 \Rightarrow 2(l - 1) = 3k$$

$$\Rightarrow 2 \mid 3k \Rightarrow 2 \mid k \quad (\because 2 \nmid 3)$$

$$\Rightarrow a \equiv 6k' + 2 \Rightarrow a \equiv 2 \pmod{6}$$

③ Read Examples 4.8, 4.10, and 4.11 from the book