Assignment for Tuesday 2/6

(I) Exercises from the book:

Section 3.2 → 4, 7, 8, 9a, 12

Section 3.3 → 18, 21a, 22, 26a

(II) Supplementary Exercises:

(13) Prove that there are infinitely many primes of the form $4k+1$.

(14) Find the smallest integer divisible by 2 and 3 which is simultaneously a square and a fifth power.

($n \in \mathbb{Z}^+$ is a k^{th} power means there exists $m \in \mathbb{Z}^+$ s.t. $n = m^k$; square means 2^{nd} power)

(15) Prove that

$$\frac{(\operatorname{lcm}(a,b,c))^2}{\operatorname{lcm}(a,b)\operatorname{lcm}(b,c)\operatorname{lcm}(c,a)} = \frac{(\gcd(a,b,c))^2}{\gcd(a,b)\gcd(b,c)\gcd(c,a)}$$

(Hint: Use FTA to find a formula for $\operatorname{lcm}(a,b,c)$ & $\gcd(a,b,c)$)

(III) Optional Exercises:

(6) Derive a formula for the number of quadruples (a,b,c,d) such that

$$\begin{aligned} 3^r 7^s &= \operatorname{lcm}(a,b,c) \\ &= \operatorname{lcm}(b,c,d) \\ &= \operatorname{lcm}(c,d,a) \\ &= \operatorname{lcm}(d,a,b) \end{aligned}$$

for $r, s \in \mathbb{Z}^+$