

According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic.
2. Being able to substitute numbers in 'formulas'.
3. Given 'formulas', being able to get other 'formulas'.
4. Being able to understand hypotheses and conclusions of theorems.
5. Being able to understand the proofs of theorems, step by step.
6. Being able to really understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems.
7. Being able to generalize and extend theorems.
8. Being able to see new relationships and discover and prove entirely new theorems.

'Formulas' can also mean elementary properties of concepts.

Basic Mathematical Ability for Math majors.

Non-trivial applications of Math

Research in Mathematics

Calculus courses focus on a mixture of 1 & 2.

Math 230 focuses on 3 & 4.

Math 332 focuses on 3 & 4 with some of 5.

In this course, the focus is more on the upper part of 3, 4, 5 and 6. The first half of the course focused on 3, 4, and 5 using elementary properties of divisibility (and congruences). In the second half, the focus has shifted away from 3 and 4, to more towards 5 and 6.

The aim is to give you a firm foundation in the levels up to 6, so that you can go on to 7 & 8, both as mathematicians and engineers (a lot of engineering is deep applied mathematics).

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This course has given you a foundation for both higher level courses in number theory, and applications of number theory to cryptography. Math 430 (Applied Algebra) would complete your preparation in this regard.

Of course, ~~and~~ many of you might not be interested in further studies in cryptography or number theory. For these students, and everybody else, I hope this course has helped you make progress on ^{the} levels of mathematical understanding and mathematical maturity (as described on the previous page). I would consider this a successful course if you have gained confidence in your ability to read, understand, and write proofs, especially as compared to the beginning of the semester.

I hope to see many of you in my courses (Math 454 and Math 435) next year.

Best wishes,
Hemanshu