Math 435/535 HW #1

Due Thursday, Jan 31st.

BT refers to Bertsimas & Tsitsiklis (the course textbook).

① BT 1.9

② BT 1.14ac

③ BT 1.13

④ Supplementary Problem #1:
Convert the following LP into standard form

$$\begin{aligned} \min \quad & 2x_1 - 3x_2 + x_3 \\ \text{s.t.} \quad & -x_1 + 4x_2 - 3x_3 = -2 \\ & 3x_1 + 4x_2 + x_3 \le 4 \\ & 2x_1 - 5x_3 \ge 1 \\ & x_2 \le 0 \\ & x_3 \ge 0 \end{aligned}$$

Supplementary Problem #2:

⑤ 1 Consider the following two optimization problems

$$\left.\begin{aligned} \min \quad & \frac{c^T x + d}{e^T x + f} \\ \text{s.t.} \quad & Ax = b \\ & e^T x + f > 0 \end{aligned}\right\} \text{— ①}$$

$$\left.\begin{aligned} \min \quad & c^T y + dz \\ \text{s.t.} \quad & Ay - bz = 0 \\ & e^T y + fz = 1 \\ & z \ge 0 \end{aligned}\right\} \text{— ②}$$

Note: $x, c, e \in \mathbb{R}^n$, A is $m \times n$, $b \in \mathbb{R}^m$, $d, f \in \mathbb{R}$. $y \in \mathbb{R}^n$, $z \in \mathbb{R}$.

Under the assumption that the feasibility sets are non-empty, show ① and ② are equivalent as follows —

a) Let x be a feasible solution of ①, then construct a solution (y, z) of ② and prove that (y, z) is feasible in ② and has the same objective function value.

b) Let (y, z) be a feasible solution of ②, then

for $z \neq 0$, construct a solution x of ① and prove that it is feasible in ① and has the same objective function value.

(Optional: If $z = 0$, what can we do?)

Caution: Show your steps and proofs explicitly. For example, in ⑤a), define y and z carefully in terms of x. Then, explicitly show why this y & z satisfy the constraints of ② and give the same value for the objective function. Similarly in ⑤b).

Don't hesitate to ask for help if you need any.