MATH 435 / 535 HW #4

(1) Consider the following LP

$$\begin{aligned} \max \quad & 3x_1 + 4x_2 \\ \text{s.t.} \quad & 2x_1 + 5x_2 \leq 20 \\ & 4x_1 + 3x_2 \leq 24 \\ & x_1 + x_2 \geq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(a) Sketch the feasible set. Identify all the corners of the feasible region (give their x_1, x_2 coords). Name them by A, B, C, ...

(b) Write the LP in standard form.

(c) What is the rank of the matrix A that you get?

(d) Find the basic feasible solutions of this Standard-form LP that correspond to each corner point identified in part (a). You should clearly identify the basis B for each corner point and then solve $Bx_B = b$ to get the BFS corresponding to that B.

(e) Do one iteration of the simplex method (5 steps described at the end of the class) starting from any one of the BFS found in (d).

(f) Now add the fourth constraint $14x_1 + 7x_2 \leq 76$. Is there a degenerate BFS now? Why? If yes, then identify the corresponding corner point of the feasible region. Show that there are more than $n-m$ x_j's set to zero at this basic solution.

(2) (a) Let x_j be a non-basic variable in a BFS x. Show that the reduced cost $\overline{c_j}$ of x_j is

$$\overline{c_j} = c^T d \quad \text{where } d \text{ is } j^{th} \text{ basic direction.}$$

[Note x is a BFS of the LP

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

]

(b) BT 3.2(a) [Hint: you can give a very short proof using part (a). It would be useful to think of a direct proof as well.]

(3) BT 3.5

Also, think about BT 3.1 and BT 3.2(b).

This shows why we only need to find a locally optimal solution of a LP