MATH 435/ 535 : Homework #1

Do all the following problems. Due Thursday, 1/21.

All problems require explicit and detailed proofs. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

Below 'BT x.y' refers to the corresponding exercise in the course textbook.

1. BT 1.9

2. BT 1.14ac

3. BT 1.13

4. *Supplementary Problem #1*: Convert the following LP into the standard form

$$\begin{array}{lrl}
\min & 2x_1 - 3x_2 + x_3 & \\
\text{s.t.} & & \\
& -x_1 + 4x_2 - 3x_3 & = -2 \\
& 3x_1 + 4x_2 + x_3 & \leq 4 \\
& 2x_1 - 5x_3 & \geq 1 \\
& x_2 & \leq 0 \\
& x_3 & \geq 0
\end{array}$$

5. *Supplementary Problem #2*: Consider the following two optimization problems where $x, c, e, y \in \mathbb{R}^n$, A is $m \times n$, $b \in \mathbb{R}^m$, and $d, f, z \in \mathbb{R}$,

$$\begin{array}{llrl}
P1: & \min & \frac{c^T x + d}{e^T x + f} & \\
& \text{s.t.} & & \\
& & Ax & = b \\
& & e^T x + f & > 0
\end{array}$$

$$\begin{array}{llrl}
P2: & \min & c^T y + dz & \\
& \text{s.t.} & & \\
& & Ay - bz & = 0 \\
& & e^T y + fz & = 1 \\
& & z & \geq 0
\end{array}$$

Under the assumption that their respective feasibility sets are non-empty, show that $P1$ and $P2$ are equivalent using the steps below. (Caution: Prove explicitly that a solution is feasible and has a given objective function value.)

(a) Let x be a feasible solution of $P1$. Then construct a feasible solution (y, z) of $P2$ that has the same objective function value.

(b) Let (y, z) be a feasible solution of $P2$. Then for $z \neq 0$, construct a feasible solution x of $P1$ that has the same objective function value. (Optional: If $z = 0$, what can we do?)