MATH 453 : Homework #2

Do all the following problems. Due Thursday, 1/28.

All problems require explicit and detailed statements, especially the ones requiring a combinatorial proof. Include enough detail in your solutions so that your explanation is convincing to someone who hasn't thought about the problem before. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

Section 2.8: 3, 4, 5, 8.

Section 2.11: 6, 10, 11.

Section 2.13: 3, 7, 9.

Section 2.14: 3, 5 and prove it like Theorem 2.7, 8, 16be, 17a.

Section 2.19: 1a, 2a, 7, 8, 9.

Supplementary Problem 1: Give combinatorial proofs for two of the following identities:

(a) $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$

(b) $\binom{k}{l}\binom{n}{k} = \binom{n}{l}\binom{n-l}{k-l}$

(c) $\sum_{k=r}^{n} \binom{k}{r} = \binom{n+1}{r+1}$

Supplementary Problem 2: Find and prove a formula for $\sum_{r,s,t\geq 0: r+s+t=n} \binom{m_1}{r}\binom{m_2}{s}\binom{m_3}{t}$.

Supplementary Problem 3: Given five points in the plane with integer coordinates, prove that the midpoint of the line segment joining some pair of them also has integer coordinates.

Supplementary Problem 4: Given any set of ten positive integers all strictly between 0 and 100, prove that there are two disjoint nonempty subsets of this set with equal sums of their elements.