MATH 554 : Homework #5

Problems 1 and 2 are compulsory. Solve two out of the remaining three problems.
Due Monday, April 11th.

All problems require explicit and detailed proofs. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

1. Prove that: Let p be prime and L be a set of non-negative integers. If $\mathcal{F}$ is a p-modular L-intersecting family of subsets of $[n]$, where $|L| = s$, then $|\mathcal{F}| \leq \sum_{i=0}^{s} \binom{n}{i}$.

2. Let $n \geq 2s$, and L be a set of s non-negative integers. Prove that every L-intersecting k-uniform family of subsets of $[n]$ has size at most $\binom{n}{s}$.

3. Let $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ be subsets of $[n]$. If $|A_i \cap B_i|$ is odd for all i and $|A_i \cap B_j|$ is even for $i < j$, then $m \leq n$, and this is sharp.

4. Let $q = p^k$, where p is prime. If $A_1, \ldots, A_m$ are subsets of $[n]$ such that each $|A_i|$ is not divisible by q, but each $|A_i \cap A_j|$ is divisible by q, then $m \leq n$.

5. For $n, p \in \mathbb{N}$ with p prime and $n > 2p$, if $G_{n,p}$ is the graph whose vertices are the incidence vectors of $(2p-1)$-sets in $[n]$, with two vertices adjacent when their Euclidean-distance in $\mathbb{R}^n$ is $\sqrt{2p}$, then $\chi(G_{n,p}) \geq \binom{n}{2p-1} / \sum_{i=0}^{p-1} \binom{n}{i}$.