MATH 435/ 535 : Homework #2

Do all the following problems. Due Thursday, 1/26, in class before the lecture starts.

All problems require explicit and detailed proofs. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Re-read the "'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

Below 'BT x.y' refers to the corresponding exercise in the course textbook.

1. BT 1.4

2. BT 1.5ab

Comment: In 1.5b, proving the following two statements is enough to finish the proof. You need to submit the proof of only ONE of these two statements. Below, P0 refers to the given optimization problem, P1 refers to the first reformulation, and P2 refers to the second reformulation.

(i) Given a feasible solution of P0, find a feasible solution of P1 with cost at most the cost in P0. And, given a feasible solution of P1, find a feasible solution of P0 with cost at most the cost in P1.

(ii) Given a feasible solution of P0, find a feasible solution of P2 with cost at most the cost in P0. And, given a feasible solution of P2, find a feasible solution of P0 with cost at most the cost in P2.

For the following two problems, assume that we have already proved: Every non-empty polyhedron in the standard form, $\mathcal{P} = \{x \in \mathbb{R}^n | Ax = b, x \ge 0\}$ has at least one BFS (extreme point/vertex).

3. BT 2.6

4. BT 2.4, and construct a non-empty polyhedron with no extreme points. *Hint:* Think about how the feasible set changes (geometrically) when an LP is converted into the standard form.