MATH 435/ 535 : Homework #4

Do all the following problems. Due Thursday, 2/9, in class before the lecture starts.

All problems require explicit and detailed proofs. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Re-read the " 'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

Below 'BT x.y' refers to the corresponding exercise in the course textbook.

1. BT 2.9

2. (a) Let x_j be a non-basic variable in a BFS x of a standard form LP $\min\{c^T x | Ax = b,\ x \geq 0\}$. Show that the reduced cost $\overline{c_j}$ of x_j is $\overline{c_j} = c^T d$ where d is the jth basic direction.

(b) Use part (a) to prove BT 3.2a.

Hint: In the backward implication for part (b), any other feasible solution y can be expressed as $y = x + 1(y - x)$. Use this to show $c^T x \leq c^T y$.

3. BT 3.5

4. Consider the following LP

$$\begin{array}{lrl} \max & 3x_1 + 4x_2 & \\ \text{s.t.} & 2x_1 + 5x_2 & \leq 20 \\ & 4x_1 + 3x_2 & \leq 24 \\ & x_1 + x_2 & \geq 2 \\ & x_1 & \geq 0 \\ & x_2 & \geq 0 \end{array}$$

(a) Sketch the feasible set. Identify all the corners of the feasible region (give their x_1, x_2 coordinates). Name them by A, B, C....

(b) Write the LP in standard form.

(c) What is the rank of the matrix A you get?

(d) Find the basic feasible solutions of the standard form LP that correspond to each corner identified in part (a). You should clearly identify the basis B for each corner point and then solve $Bx_B = b$ to get the BFS corresponding to that B.

(e) Do one iteration of the Simplex Algorithm (5 steps given in class) starting from any one of the BFS you found in part (d).

Comment: I will finish discussing all the five steps of the Simplex method on Tuesday, so you might have to postpone completing part (e) till Tuesday.

(f) Now add the fourth constraint $14x_1 + 7x_2 \leq 76$ to the LP. Is there a degenerate BFS now? Why? If yes, then identify the corresponding corner point of the feasible region. Show that there are more than $n - m$ variables x_j set to zero at this basic solution.