e.g. max $x_1 + 4x_2 + x_3$

s.t. $2x_1 + 5x_2 + x_3 + x_4 = 7 - 3\theta$

$x_1 + 3x_2 + x_3 + x_5 = 5 - 2\theta$

$x_1 \geq \theta, \ x_2 \geq 0, \ 0 \leq x_3 \leq 3 + 2\theta, \ x_4, x_5 \geq 0$

Find a basic solution:

(Put $\theta = 0$ & see what happens —

Apply simplex to get $[0 \ 1 \ 2 \ 0 \ 0]$ as the optimal soln. with x_2 & x_3 as basic variables)

Let x_1, x_4, x_5 be non-basic, so put them equal to their lower bounds & then solve for x_2 & x_3 as basic variables

$x_1 = \theta \quad x_4 = 0, \ x_5 = 0$

$$\left.\begin{aligned} 5x_2 + x_3 &= 7 - 5\theta \\ 3x_2 + x_3 &= 5 - 3\theta \end{aligned}\right\} \Rightarrow \begin{aligned} x_2 &= 1 - \theta \\ x_3 &= 2 \end{aligned}$$

① $\rightarrow$ $[\theta \ \ 1-\theta \ \ 2 \ \ 0 \ 0]$ is a family of basic solutions

Each of these is dual-feasible for all θ. $\left(\begin{array}{l} c^T - c_B^T B^{-1} A \geq 0^T \\ \text{i.e., } p^T A \leq c^T \end{array} \right)$

Also, this is primal feasible iff $1 - \theta \geq 0$ and $2 \leq 3 + 2\theta$

i.e. $-0.5 \leq \theta \leq 1$

$\therefore$ ① is an optimal soln. if $-0.5 \leq \theta \leq 1$

Case 1 Increase θ to just above 1

Now ① is no longer primal-feasible however, apply dual-simplex starting from ① (dual BFS).

Then, x_2 leaves the basis & the resulting dual simplex iteration gives

(2) → $[\theta \;\; 0 \;\; 7-5\theta \;\; 0 \;\; -2+2\theta]$

which is a dual BFS

This is primal feasible $\Leftrightarrow 0 \leq 7-5\theta \leq 3+2\theta$

$\& \;\; -2+2\theta \geq 0$

i.e., $1 \leq \theta \leq 1.4$

$\therefore$ (2) is an optimal soln. when $1 \leq \theta \leq 1.4$

Case 1a Increase θ above 1.4

So, (2) is no longer primal feasible, but it is dual BFS

In the dual simplex iteration, x_3 can exit the basis but no var. can enter the basis,

so,

The LP is infeasible when $\theta > 1.4$.

Case 2 Decrease θ below -0.5

So, (1) is no longer primal feasible, but it is dual BFS

In the dual simplex iteration, x_3 exits & we get

(3) → $[\theta \;\; 0.8-1.4\theta \;\; 3+2\theta \;\; 0 \;\; -0.4-0.8\theta]$

is the new dual BFS

This is primal feasible iff $0.8-1.4\theta \geq 0$

$\& \;\; -0.4-0.8\theta \geq 0$

i.e. $\theta \leq -0.5$

$\therefore$ (3) is an optimal soln. when $\theta \leq -0.5$