- Read & Understand Example 3.5 (Full Tableau Example)
  Example 3.6 gives a pivoting rule and an LP with cycling

- Another example

$$\begin{aligned} \min\ & -8x_1 - 9x_2 - 5x_3 \\ \text{s.t.}\ & x_1 + x_2 + 2x_3 \le 2 \\ & 2x_1 + 3x_2 + 4x_3 \le 3 \\ & 6x_1 + 6x_2 + 2x_3 \le 8 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Std. form

$$\begin{aligned} \min\ & -8x_1 - 9x_2 - 5x_3 \\ \text{s.t.}\ & x_1 + x_2 + 2x_3 + x_4 = 2 \\ & 2x_1 + 3x_2 + 4x_3 + x_5 = 3 \\ & 6x_1 + 3x_2 + 2x_3 + x_6 = 8 \\ & x \ge 0 \end{aligned}$$

$m = 3, n = 6$

A is $m \times n$ & has rank $= 3$ (lin. ind. cols)

A_4, A_5, A_6 form the initial basis & x_4, x_5, x_6 are the basic vars

		x_1	x_2	x_3	x_4	x_5	x_6
	0	-8	-9	-5	0	0	0
x_4	2	1	1	2	1	0	0
← x_5	3	2	(3)	4	0	1	0
x_6	8	6	3	2	0	0	1

- Now need to make column 2 into $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ & $\bar{c}_2 = 0$ by row operations

	9	-2	0	7	0	3	0
x_4	1	1/3	0	2/3	1	-1/3	0
x_2	1	2/3	1	4/3	0	1/3	0
← x_6	2	(2)	0	-6	0	-2	1

- Now need to make column 1 in $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ & $\bar{c}_1 = 0$ by row operations

	11	0	0	1	0	1	1
x_4	2/3	0	0	5/3	1	0	-1/6
x_2	1/3	0	1	10/3	0	+1	-1/3
x_1	1	1	0	-3	0	-1	1/2

No negative reduced costs

∴ solution is optimal with cost = -11

$$x_1 = 1 \;,\; x_2 = 1/3 \;,\; x_3 = 0$$

The value of slack variables is

$$x_4 = \frac{2}{3} \;,\; x_5 = 0 \;,\; x_6 = 0$$

- Another example

$$\begin{aligned} \min \quad & -2x_1 - 3x_2 - x_3 - x_4 \\ \text{s.t.} \quad & x_1 - x_2 - x_3 + x_5 = 2 \\ & -2x_1 + 5x_2 - 3x_3 + 3x_4 + x_6 = 10 \\ & 2x_1 - 5x_2 + 3x_4 + x_7 = 5 \\ & x \geq 0 \end{aligned}$$

A_5, A_6, A_7 form the initial basis

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	0	-2	-3	-1	-1	0	0	0
x_5	2	1	-1	-1	0	1	0	0
← no choice x_6	10	-2	(5)	-3	3	0	1	0
x_7	5	2	-5	0	3	0	0	1

- Make column 2 into $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ & $\bar{c}_2 = 0$

no choice	6	$-16/5$	0	$-14/5$	$-14/5$	0	$3/5$	0
← x_5	4	$3/5$	0	$-8/5$	$-3/5$	1	$1/5$	0
x_2	2	$-3/5$	1	$-3/5$	$+3/5$	0	$1/5$	0
x_7	15	0	0	-3	0	0	1	1

- Make column 1 into $\begin{bmatrix}1\\0\\0\end{bmatrix}$ & $\bar{C}_1 = 0$

	$82/3$	0	0	$-34/3$	-6	$16/3$	$5/3$	0
x_1	$20/3$	1	0	$-8/3$	-1	$5/3$	$1/3$	0
x_2	$14/3$	0	1	$-5/3$	1	$2/3$	$1/3$	0
x_7	15	0	0	-3	0	0	1	1

we found column j with $\bar{C}_j < 0$ but all components of the corresponding u are negative ($= B^{-1}A_j$)

i.e., we found a basic direction that is feasible & allows us to make as large a step we want

i.e., the optimal cost is $-\infty$