Another Proof of "$\sqrt{2}$ is irrational"

Fermat's method of infinite descent:

This is disguised form of getting a contradiction to the Well Ordering Principle by producing an infinite sequence of decreasing positive integers if $\sqrt{2}$ is assumed to be rational.

Suppose $\sqrt{2} = \frac{a}{b}$

Let $a_1 = 2b - a$ & $b_1 = a - b$

then $\frac{a}{b} = \frac{a_1}{b_1}$ $\quad (\because a^2 = 2b^2)$

Also, $0 < b_1 < b$

since $1 < \frac{a}{b} < 2 \Rightarrow b < a < 2b \Rightarrow 0 < a - b < b$

Now, define $a_{k+1} = 2b_k - a_k$ & $b_{k+1} = a_k - b_k$, $k \geq 1$

& by same reasoning we get $\sqrt{2} = \frac{a}{b} = \frac{a_{k+1}}{b_{k+1}}$

and $0 < b_{k+1} < b_k < b$, $k \geq 1$

Giving an infinite decreasing sequence of positive integers $\langle b_k \rangle_{k=1}^{\infty}$ which is not possible.

Question: Use this idea to prove that $\sqrt{N}$ is irrational where N is not a square

(How will you define a_k & b_k in this case?)