## Some Examples (Section 4.4)

(1) Solve the system of congruences

$$17x \equiv 3 \pmod{2}$$
$$17x \equiv 3 \pmod{3}$$
$$17x \equiv 3 \pmod{5}$$
$$17x \equiv 3 \pmod{7}$$

Soln.

Simplify:

$17x \equiv 3 \pmod{2} \Leftrightarrow 17x = 2k+3 \Leftrightarrow x = 2(k-8x+1)+1 \Leftrightarrow x \equiv 1 \pmod{2}$

$17x \equiv 3 \pmod{3} \Leftrightarrow 17x = 3l+3 \Leftrightarrow 2x = 3(l-5x+1)+0 \Leftrightarrow 2x \equiv 0 \pmod{3}$

$17x \equiv 3 \pmod{5} \Leftrightarrow 17x = 5m+3 \Leftrightarrow 2x = 5(m-3x)+3 \Leftrightarrow 2x \equiv 3 \pmod{5}$

$17x \equiv 3 \pmod{7} \Leftrightarrow 17x = 7n+3 \Leftrightarrow 3x = 7(n-2x)+3 \Leftrightarrow 3x \equiv 3 \pmod{7}$

Also, note that $2x \equiv 0 \pmod{3} \Leftrightarrow x \equiv 0 \pmod{3}$
since $\gcd(2,3)=1$ we can cancel 2.
and $3x \equiv 3 \pmod{7} \Leftrightarrow x \equiv 1 \pmod{7}$
since $\gcd(3,7)=1$ we can cancel 3.

Finally, $x \equiv 4 \pmod{5} \Leftrightarrow x = 5k+4 \Rightarrow 2x = 5k'+8$
$\Rightarrow 2x = 5k''+3$
$\Leftrightarrow 2x \equiv 3 \pmod{5}$

Thus, it is enough to solve the system

$$x \equiv 1 \pmod{2}$$
$$x \equiv 0 \pmod{3}$$
$$x \equiv 4 \pmod{5}$$
$$x \equiv 1 \pmod{7}$$

Which can be done either by using CRT or by direct substitutions.

(2) The system $a \equiv 0 \bmod 2$, $a \equiv 2 \bmod 3$, $a \equiv 2 \bmod 6$ is equivalent to the smaller system $a \equiv 0 \bmod 2$, $a \equiv 2 \bmod 3$

because:

$$\left.\begin{array}{l} a \equiv 0 \bmod 2 \Rightarrow a = 2l \\ a \equiv 2 \bmod 3 \Rightarrow a = 3k+2 \end{array}\right\} \Rightarrow 2l = 3k+2 \Rightarrow 2(l-1) = 3k$$
$$\Rightarrow 2 \mid 3k \Rightarrow 2 \mid k \quad (\because 2 \nmid 3)$$
$$\Rightarrow a \equiv 6k'+2 \Rightarrow a \equiv 2 \bmod 6$$

(3) Read Examples 4.8, 4.10, and 4.11 from the book