Some Examples (section 6.1)

① Example for proof of Thm. 6.2 (b)

$n = 180 = 2^2 \cdot 3^2 \cdot 5$

$(1+2+2^2)(1+3+3^2)(1+5)$
$= 1+2+3+2^2+5+2\cdot3+3^2+2\cdot5+2^2\cdot3+$
$+3\cdot5+2\cdot3^2+2^2\cdot5+2\cdot3\cdot5+2^2\cdot3^2+3^2\cdot5+2^2\cdot3\cdot5$
$+2\cdot3^2\cdot5+2^2\cdot3^2\cdot5$
$= 1+2+3+4+5+6+9+10+12+15+18+20+30+36$
$+45+60+90+180 \quad = 546$
$= \sigma(180)$

same answer

by Thm., $= \left(\frac{2^3-1}{2-1}\right)\left(\frac{3^3-1}{3-1}\right)\left(\frac{5^2-1}{5-1}\right) = 7\cdot13\cdot6 = 546$

These are all the devisors of 180

② Let n be square-free number
Then $n = p_1 p_2 \cdots p_r$ for distinct primes p_i

Thus $\sigma(n) = (1+1)(1+1)\cdots(1+1) = 2^r$, by Thm

③ If $n = 2^{k-1}$ Then $\sigma(n) = 2n-1$

Soln.
$\sigma(n) = \sigma(2^{k-1}) = 2^k - 1 = 2\cdot2^{k-1} - 1 = 2n-1$

by Thm

④ For a prime number p, $\sigma(p) = 1+p$, by Thm.

⑤ If p & $p+2$ are twin primes, Then $\sigma(p+2) = \sigma(p)+2$

Soln.
$\sigma(p+2) = 1+(p+2) = (1+p)+2 = \sigma(p)+2$

since $p+2$ is prime — since p is prime

⑥ $f(n) = n^k$ is a multiplicative function

Soln. $f(nm) = (nm)^k = n^k m^k = f(m)f(n)$ (we didn't even need $\gcd(m,n)=1$)

⑦ If two multiplicative functions f, g agree on powers of primes, Then they agree on all integers.

since f & g take equal value at each power of a prime

prime factorization of n

$f(n) = f(p_1^{k_1}\cdots p_r^{k_r}) = f(p_1^{k_1})f(p_2^{k_2})\cdots f(p_r^{k_r}) = g(p_1^{k_1})\cdots g(p_r^{k_r})$
$= g(p_1^{k_1}\cdots p_r^{k_r}) = g(n)$