Do all the following problems. Due Wednesday, 2/19, in class before the lecture starts.

All problems require explicit and detailed proofs. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Re-read the “‘Why and How’ of Homework” section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don’t hesitate to ask. You can contact me during office hours, or through email.

Below ‘BT x.y’ refers to the corresponding exercise in the course textbook.

1. BT 3.18ab

2. (a) Let \( x_j \) be a non-basic variable in a BFS \( x \) of a standard form LP \( \min \{ c^T x | Ax = b, \ x \geq 0 \} \). Show that the reduced cost \( \overline{c}_j \) of \( x_j \) is \( \overline{c}_j = c^T d \) where \( d \) is the \( j \)th basic direction.

(b) Use part (a) to prove BT 3.2a.

*Hint:* In the backward implication for part (b), any other feasible solution \( y \) can be expressed as \( y = x + 1(y - x) \). Use this to show \( c^T x \leq c^T y \).

3. BT 3.5

4. Consider the following LP

\[
\begin{align*}
\text{max} & \quad 3x_1 + 4x_2 \\
\text{s.t.} & \quad 2x_1 + 5x_2 \leq 20 \\
& \quad 4x_1 + 3x_2 \leq 24 \\
& \quad x_1 + x_2 \geq 2 \\
& \quad x_1 \geq 0 \\
& \quad x_2 \geq 0
\end{align*}
\]

(a) Sketch the feasible set. Identify all the corners of the feasible region (give their \( x_1, x_2 \) coordinates). Name them by A, B, C,...

(b) Write the LP in standard form.

(c) What is the rank of the matrix \( A \) you get?

(d) Find the basic feasible solutions of the standard form LP that correspond to each corner identified in part (a). You should clearly identify the basis \( B \) for each corner point and then solve \( Bx_B = b \) to get the BFS corresponding to that \( B \).

(e) Do one iteration of the Simplex Algorithm (5 steps given in class) starting from any one of the BFS you found in part (d).

(f) Now add the fourth constraint \( 14x_1 + 7x_2 \leq 76 \) to the LP. Is there a degenerate BFS now? Why? If yes, then identify the corresponding corner point of the feasible region. Show that there are more than \( n - m \) variables \( x_j \) set to zero at this basic solution.
Comment: You might want to postpone the following computational problem till
Monday, when I will show you an example of simplex Algorithm applied to tableau
form. IF you want to get started on this problem earlier, then first read Example 3.5
on page 101.

5. Consider the following Simplex Tableau:

\[
\begin{array}{ccccccc}
 & 0 & -2 & -3 & 1 & 12 & 0 & 0 \\
x_5 & 0 & -2 & -9 & 1 & 9 & 1 & 0 \\
x_6 & 0 & \frac{1}{3} & 1 & -\frac{1}{3} & -2 & 0 & 1 \\
\end{array}
\]

Note that \(x_5\) and \(x_6\) are the current Basic variables.

(a) Apply the Simplex tableau method with the following pivoting rules:

The non-basic variable with the most negative cost enters the Basis, and

In case of ties for the leaving variable, choose the variable corresponding to the row that

is higher up in the tableau.

Demonstrate that the Simplex method cycles (i.e., you get the exact same Tableau, and conse-

quently the same Basis, after a few iterations).

(b) Apply the Simplex method with Bland’s rule and demonstrate that the Simplex method
does not cycle by finding the optimal solution.