MATH 486/ 522 : Homework #5

Do all the following problems. Due Wednesday, 2/19, in class before the lecture starts.

Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Re-read the " 'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

1. Section 7.1: #4.

2. Section 8.1: Solve one of (#3 or #6).

3. Section 8.1: #7 (First, show how to set up this problem as graph coloring problem - define the vertices, edges, colors, etc.).

4. Section 8.5: Solve one of (#1ab or #4).

5. Math 486 solve one of the following two parts; Math 522 solve both parts:
(a) We want to restrict a variable x to take values in a set $\{a_1, \ldots, a_m\}$. Show how to do this using two linear constraints.

(b) We are given two linear constraints $\mathbf{a}^T\mathbf{x} \geq b$ and $\mathbf{c}^T\mathbf{x} \geq d$ in which each component of both $\mathbf{a}$ and $\mathbf{c}$ is nonnegative. (Note that $\mathbf{a}^T\mathbf{x}$ means $\sum_{i=1}^{n} a_i x_i$, similarly $\mathbf{c}^T\mathbf{x}$.) The variable vector $\mathbf{x}$ is required to satisfy **at least** one of the these two constraints. Show how to rewrite these linear constraints so that this requirement is satisfied. (Comment: You should not force $\mathbf{x}$ to satisfy both constraints always, there should be a choice of either satisfying only constraint number 1, or satisfying only constraint number 2, or satisfying both constraints.)

6. Let $G = (V(G), E(G))$ denote a directed graph (network) we are building for transporting a commodity between the vertices in $V(G)$. We are given, for each $i \in V(G)$, a supply or demand $b_i \in \mathbb{Z}$ for the commodity, such that $\sum_{i \in V} b_i = 0$. We are also given for each edge (i, j): u_{ij}, the maximum capacity of edge from i to j; c_{ij}, the transportation cost of sending one unit of the commodity from vertex i to j; and d_{ij}, the building cost of constructing an edge (transportation link) from vertex i to j. We would like to build such a network (that is decide which edges to build, and decide how much flow of commodity to send on each edge) so that total building and transportation costs are minimized, and all demand is met. Formulate this problem as a mixed integer linear program.
(Comment: At vertex i: if $b(i) = 0$ then it means difference between inflow and outflow of commodity at i must equal 0; if $b(i) > 0$ this means vertex i is a factory/producer and the difference between inflow and outflow of commodity at i must equal supply value b_i; if $b(i) < 0$ this means vertex i is a consumer and the difference between inflow and outflow of commodity at i must equal supply value b_i.)