Math 486/522 – Introduction to Mathematical Modeling

Course Description from Bulletin:  This course provides an introduction to problem-driven (as opposed to method-driven) applications of mathematics with a focus on design and analysis of models using tools from all parts of mathematics. (3-0-3) (C)


Prerequisites: CS 104, MATH 251, MATH 252 (concurrent), MATH 332

Objectives:
1. Students will develop an understanding of applied mathematics as a thought-process and a toolbox for the study of real-world phenomena from engineering, natural and social sciences.
2. Students will learn concepts and tools from different parts of mathematics – continuous, discrete, and probabilistic – as they are applied to build and refine models for various applications.
3. Students will study how to compare the modeling results to observations and how models can be improved.
4. Students will do a 8-10 week long project where they apply the modeling process to analyze an open ended real-life problem, with a deliverable of a project report and programming implementation.
5. Students will develop good habits for understanding, communicating, and writing mathematical knowledge through classroom participation, homework, and projects.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:
1. Discrete change in financial and biological population systems – Difference equations and discrete dynamical systems, solutions and stability 5
2. Physical models – Proportionality and geometric similarity 3
3. Model fitting – Errors, Chebyshev criterion, least squares criterion, linear regression, and data transformation 5
4. Discrete optimization models – Linear optimization, geometric and algebraic solutions, integer programs and combinatorial optimization, binary decisions 3
5. Network models – Graphs and networks, network flows, assignment problems, graph coloring, vertex covers, local search algorithms 5
6. Discrete probabilistic models – Finite discrete time Markov chains and stationary distribution, component and system reliability 2
7. Simulation Modeling – Monte Carlo algorithms, random point generation, queuing models 3
8. Population models – Ordinary differential equations, equilibria, phase diagrams and solutions fields 4
9. Competing species and predator-prey models – Dynamical systems, Euler’s method, solving linear dynamical systems 5
10. Continuous optimization models – Multivariable optimization, gradient method, Lagrange multipliers, Newton’s method 3
11. Special topics – e.g., complex network models, game theoretic models

Assessment:

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<th>Assessment</th>
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<tbody>
<tr>
<td>Homework</td>
<td>15-25%</td>
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<tr>
<td>Semester Project</td>
<td>20-30%</td>
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<tr>
<td>Mid-Term Exams</td>
<td>20-30%</td>
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<td>Final Exam</td>
<td>20-30%</td>
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