MATH 554 - Discrete Applied Mathematics II

Course Description from Bulletin: A graduate-level course that introduces students in applied mathematics, computer science, natural sciences, and engineering, to the application of modern tools and techniques from various fields of mathematics to existential and algorithmic problems arising in discrete applied math. Probabilistic methods, entropy, linear algebra methods, combinatorial nullstellensatz, and Markov chain Monte Carlo, are applied to fundamental problems like Ramsey-type problems, intersecting families of sets, extremal problems on graphs and hypergraphs, optimization on discrete structures, sampling and counting discrete objects, etc. (3-0-3)

Enrollment: Graduate Elective Course for Applied Math and other majors.

Textbook(s): Lecture Notes, supplemented by recommended textbooks: S. Jukna, *Extremal Combinatorics with Applications to Computer Science*; N. Alon and J. Spencer, *Probabilistic Method, 3rd. ed.*; O. Haggstrom, *Finite Markov Chains and Algorithmic Applications*.

Other required material:

Prerequisites: MATH 332 and (MATH 474 or MATH 475) and (MATH 454 or MATH 553); or instructor's consent based on familiarity with linear algebra, probability, and discrete math.

Objectives:

1. Students will be introduced to the exchange of ideas and methods between different fields of mathematics and discrete mathematics.
2. Students will learn modern methods for solving problems from various parts of graph theory, combinatorics and computer science as developed in the recent decades
3. Students will be introduced to problems and concepts that are important in contemporary discrete mathematics.
4. Students will become proficient in applying elementary probabilistic methods, the entropy method, the linear algebra method, and the Combinatorial Nullstellensatz, and students will learn the fundamentals of approximate counting through MCMC.
5. Students will work on a project topic to further explore an advanced method through an application related to their research interests.

Lecture schedule: Three 50-minute (or two 75-minute) lectures per week

Course Outline:	Hours
1. Probabilistic Method	12

a. Elementary arguments
b. Dependency Graph and Lovasz Local Lemma - symmetric and asymmetric forms
c. Binomial Random Graph Model - First and Second moment methods and threshold functions
d. Applications to graph and hypergraph coloring, set systems, Ramsey numbers, excluded subgraphs, threshold for balanced graphs, etc.

2. Shannon Entropy 7
   a. Properties of the entropy function and conditional entropy function
   b. Shearer's lemma and its alternate form
   c. Applications to intersecting families of sets and graphs, maximum number of copies of a fixed hypergraph H in the family of all hypergraphs with given number of edges, number of independent sets in a regular bipartite graph.
3. Linear Algebra Method 7
   a. Vector spaces over finite fields, vector spaces of multivariable-polynomials, dimension and spanning sets, and properties of dot product.
   b. Diagonal and Triangular criteria for linear independence of polynomials, multilinear reduction principle.
   c. Applications to k-distance sets, various forms of L-intersecting families, and p-modular L-intersecting families.
4. Combinatorial Nullstellensatz 5
   a. Number of roots of a multivariable polynomial and the Combinatorial Nullstellensatz.
   b. Applications to additive combinatorics, and existence of subgraph with specified or forbidden degrees.
5. Markov chain Monte Carlo 5-7
   a. Fundamentals of Homogeneous Discrete Time Markov Chains.
   b. Designing a HDMTC via a Gibbs sampler and the Metropolis process.
   c. Relation between exact counting, exact sampling, approximate sampling (FPAUS) and approximate counting (FPRAS).
   d. Methods for bounding the mixing time of a Markov chain.
   e. Applications to counting independent sets and proper q-colorings of a graph.
6. Project discussion and presentations, exams etc. 4-6

Assessment:	Homework	20-40%
	Exam	20-40%
	Project	10-20%
	Final Exam	30-50%

Syllabus prepared by: Hemanshu Kaul
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