MATH 554 : Homework #6

Submit four out of the five problems below.
Problems 1 and 2 are compulsory. Solve two out of the remaining three problems.
Due Tuesday, April 18th.

Before starting the HW, carefully read the HW related discussion and rules described in
`http://www.math.iit.edu/~kaul/TeachingSpr23/TeachingFiles/Math554.pdf`

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you are having trouble understanding a homework problem, I will be glad to direct you in the right direction without giving away the solution. You are encouraged to ask questions during the *Class*, through the *Campuswire Discussion Forums*, during the *Office Hours*, or through *Email to me*.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

If you discuss the problems with anyone, you are required to note their name at the top of your HW submission under a subtitle "Collaborator" or "Discussed with:".

1. Prove that: Let p be prime and L be a set of non-negative integers. If $\mathcal{F}$ is a p-modular L-intersecting family of subsets of $[n]$, where $|L| = s$, then $|\mathcal{F}| \leq \sum_{i=0}^{s} \binom{n}{i}$.

2. Let $n \geq 2s$, and L be a set of s non-negative integers. Prove that every L-intersecting k-uniform family of subsets of $[n]$ has size at most $\binom{n}{s}$.

3. Let $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ be subsets of $[n]$. If $|A_i \cap B_i|$ is odd for all i and $|A_i \cap B_j|$ is even for $i < j$, then prove that $m \leq n$, and this is sharp.

4. Let $q = p^k$, where p is prime. If $A_1, \ldots, A_m$ are subsets of $[n]$ such that each $|A_i|$ is not divisible by q, but each $|A_i \cap A_j|$ is divisible by q, then prove that $m \leq n$.

5. For $n, p \in \mathbb{N}$ with p prime and $n > 2p$, if $G_{n,p}$ is the graph whose vertices are the incidence vectors of $(2p-1)$-sets in $[n]$, with two vertices adjacent when their Euclidean-distance in $\mathbb{R}^n$ is $\sqrt{2p}$, then prove that $\chi(G_{n,p}) \geq \binom{n}{2p-1} / \sum_{i=0}^{p-1} \binom{n}{i}$.