MATH 435 & 535: Homework #1

Due Wednesday, 1/17, before 11pm via a PDF file uploaded to the Homework#1 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

MATH 535 students submit all five problems.
MATH 435 students submit Problems 1 through 4.

Below 'BT x.y' refers to the corresponding exercise in the course textbook: D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*, Athena Sc., 1997.

1. BT 1.9

2. BT 1.15

3. Supplementary Problem #1: Convert the following LP into the standard form

$$\begin{array}{ll} \min & 2x_1 - 3x_2 + x_3 \\ \text{s.t.} & \\ & \begin{array}{rl} -x_1 + 4x_2 - 3x_3 & = -2 \\ 3x_1 + 4x_2 + x_3 & \leq 4 \\ 2x_1 - 5x_3 & \geq 1 \\ x_2 & \leq 0 \\ x_3 & \geq 0 \end{array} \end{array}$$

4. Supplementary Problem #2: Consider the following two optimization problems where $x, c, e, y \in \mathbb{R}^n$, A is $m \times n$, $b \in \mathbb{R}^m$, and $d, f, z \in \mathbb{R}$,

$$
\begin{aligned}
P1: \quad \min \quad & \frac{c^T x + d}{e^T x + f} \\
\text{s.t.} \quad & \\
& Ax = b \\
& e^T x + f > 0
\end{aligned}
$$

$$
\begin{aligned}
P2: \quad \min \quad & c^T y + dz \\
\text{s.t.} \quad & \\
& Ay - bz = 0 \\
& e^T y + fz = 1 \\
& z \geq 0
\end{aligned}
$$

Under the assumption that their respective feasible sets are non-empty, show that $P1$ and $P2$ are equivalent using the steps below. (Caution: Prove explicitly that a solution is feasible and has a given objective function value.)

(a) Let x be a feasible solution of $P1$. Then construct a feasible solution (y, z) of $P2$ that has the same objective function value.

(b) Let (y, z) be a feasible solution of $P2$. Then for $z \neq 0$, construct a feasible solution x of $P1$ that has the same objective function value. (Optional: If $z = 0$, what can we do?)

5. [Math 535 ONLY] BT 1.13