MATH 454: Homework #10

Due Thursday, 4/4, before 11am via a single PDF file uploaded to the Homework#10 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Submit the following four problems.

1. True or False? Give a short proof or a counterexample as appropriate.

(a) Let G be a 2-connected graph with distinct vertices u and v. Given a u, v-path P, there exists another u, v-path Q that is internally disjoint from P.

(b) Let G be a connected graph with at least 3 vertices. Form a new graph G' from G by putting a new edge between every pair of vertices at distance 2 in the graph G. Then, G' is 2-connected.

2. Let G be a k-connected graph with at least $2k$ vertices and let A and B be disjoint sets of k vertices each. Prove that there exist k paths between A and B that are pairwise completely disjoint. [Hint: Use Expansion Lemma to modify the given graph before applying the Global Menger Theorem (Theorem 4.2.21: $\kappa(G) = \min\{\lambda(x, y) \mid x, y \in V(G)\}$)]

3. Use Local Menger's Theorem (Theorem 4.2.17) to prove König-Egerváry Theorem ($\alpha'(G) = \beta(G)$ when G is bipartite).

4. Textbook exercise 4.1.8 (only for the graph on left).