# MATH 454: Homework #11

**Due Wednesday, 4/17, before 11:59pm via a single PDF file uploaded to the Homework#11 under Assignments in the Blackboard course page.**

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

**Submit four out of the following five problems.**

**1.** Let $G$ be bipartite. Prove that $\chi(\overline{G}) = \omega(\overline{G})$.

**2.** Let $G$ be a graph with the property that every two odd cycles in $G$ have a common vertex. Prove that $\chi(G) \leq 5$.

**3.** Let $G$ be a graph on $n$ vertices. Use induction on $n$ to prove that: $\chi(G) + \chi(\overline{G}) \leq n + 1$.

**4.** Let $H_1$ and $H_2$ be two graphs with $V(H_1) = V(H_2)$. Let $G$ be the union of $H_1$ and $H_2$, i.e., $V(G) = V(H_1) = V(H_2)$ and $E(G) = E(H_1) \cup E(H_2)$. Prove that $\chi(G) \leq \chi(H_1)\chi(H_2)$. Show that this bound is sharp by constructing an example with non-trivial $H_1$ and $H_2$.

**5.** $G$ is allowed to have multi-edges in this problem.

(a) Prove that every graph $G$ is contained in a $\Delta(G)$-regular loopless graph.
(b) Let $G$ be a graph such that $\Delta(G) = 2k$. Prove that $\chi'(G) \leq 3k$ without using Vizing-Gupta theorem.