MATH 454: Homework #2

Due Wednesday, 1/24, before 11:59pm via a single PDF file uploaded to the Homework#2 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Please note:
– Next week we will discuss a characterization of Bipartite graphs: G is bipartite if and only G contains no odd cycles. You are allowed to use this statement for this HW, if useful.
– A graph is called k-regular if every vertex of that graph has degree equal to k.
– Please complete the reading HWs before attempting this HW.

Submit all the four problems stated below.

1. Let G be a k-regular graph with girth 4. Give a direct argument that G has at least $2k$ vertices. Characterize all such graphs with exactly $2k$ vertices.
Comment: "Characterize" means you need to find all such graphs; to justify that you have them all and no others. You need to prove that your listed graphs all have the required property, and that no other graphs have that property. In this particular question, the first part is easy, and the second part a little harder.

2. Determine whether Petersen graph is bipartite, and find the size of its largest independent set.

3. Prove that: a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$.

4. Let G be a 3-regular simple graph. Prove that G decomposes into claws if and only if G is bipartite.