MATH 454: Homework #3

Due Wednesday, 1/31, before 11:59pm via a single PDF file uploaded to the Homework#3 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Submit any four out of the five problems stated below.

1. Solve both of following two short questions.
(a) Let G be connected graph. Prove that: A vertex w in G is a cut-vertex if and only if there exist two vertices u and v in G such that every path between u and v passes through w.
(b) Show that every graph (containing at least one edge) has at least two vertices that are not cut-vertices.

2. Let G be a loopless graph with $\delta(G) \geq 3$. Prove that G contains an even cycle. (Recall that $\delta(G)$ denotes the minimum degree of vertices in a graph G, so $\delta(G) \geq 3$ means every vertex has degree at least 3.)

3. Let G be a connected simple graph not having P_4 or C_3 as an induced subgraph. Prove that G is a complete bipartite graph.

4. Let G be a connected graph. Suppose P and Q are any two maximum length paths in G. Prove that P and Q must have a common vertex.

5. In the graph below, find a bipartite subgraph with the maximum number of edges. Prove that this is the maximum, and determine whether yours is the only bipartite subgraph with this many edges.