MATH 435 & 535: Homework #5

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Math 535: Submit all the following problems.
Math 435: You may skip one of Problems 1,2,3.
Due Thursday, 2/15, by 11am in Blackboard. Submit a PDF file through Blackboard Assignment.

Below 'BT x.y' refers to the corresponding exercise in the course textbook: D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*, Athena Sc., 1997.

1. Consider a standard form polyhedron with constraint matrix A given by

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$$

and vector b given by $b^T = [1\ 1]$. Note $n = 3, m = 2$ here. Show that $x = (1, 0, 0)$ is a degenerate BFS of this polyhedron. Also show that there is an unique basis associated with x.
Comment: This shows that a degenerate Basic Solution need not correspond to two or more bases.

2. (a) Let x_j be a non-basic variable in a BFS x of a standard form LP $\min\{c^T x | Ax = b,\ x \geq 0\}$. Show that the reduced cost $\overline{c_j}$ of x_j is $\overline{c_j} = c^T d$ where d is the jth basic direction.

(b) Use part (a) to prove BT 3.2a.

Hint: In the backward implication for part (b), any other feasible solution y can be expressed as $y = x + 1(y - x)$. Use this to show $c^T x \leq c^T y$.

3. BT 3.5

4. Consider the following LP

$$\begin{array}{lll} \max & 3x_1 + 4x_2 & \\ \text{s.t.} & 2x_1 + 5x_2 & \leq 20 \\ & 4x_1 + 3x_2 & \leq 24 \\ & x_1 + x_2 & \geq 2 \\ & x_1 & \geq 0 \\ & x_2 & \geq 0 \end{array}$$

(a) Sketch the feasible set. Identify all the corners of the feasible region (give their x_1, x_2 coordinates). Name them by A, B, C....

(b) Write the LP in standard form.

(c) What is the rank of the matrix A you get?

(d) Find the basic feasible solutions of the standard form LP that correspond to each corner identified in part (a). You should clearly identify the basis B for each corner point and then solve $Bx_B = b$ to get the BFS corresponding to that B.

(e) [Comment: You might want to postpone this part till Monday if you wish.] Do one iteration of the Simplex Algorithm (5 steps given in class) starting from any one of the BFS you found in part (d).

(f) Now add the fourth constraint $14x_1 + 7x_2 \leq 76$ to the LP. Is there a degenerate BFS now? Why? If yes, then identify the corresponding corner point of the feasible region. Show that there are more than $n - m$ variables x_j set to zero at this basic solution.