MATH 454: Homework #6

Due Wednesday, 2/28, before 11:59pm via a single PDF file uploaded to the Homework#6 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Submit any five out of the six problems stated below.

1. Let $d_1, d_2, ..., d_n$ be a list of positive integers with $d_1 \leq d_2 \leq \ldots \leq d_n$.
Prove that this list is a degree sequence of a tree iff $\sum_{i=1}^{n} d_i = 2(n-1)$.

2. Prove or disprove:

(a) For every $k > 1$, there exists a graph with center consisting of two vertices at distance k.

(b) Every maximum length path in a tree contains the center of the tree.

3. Let G be a connected graph and T, T' be two spanning trees of G. For any $e \in E(T) - E(T')$, prove that there exists an $e' \in E(T') - E(T)$ such that $T' + e - e'$ and $T - e + e'$ are both spanning trees of G.

4. Prove that every tree with maximum degree $\Delta > 1$ has at least Δ leaves. Show this is best possible by constructing for each value of n and Δ, an n-vertex tree with exactly $\Delta = \Delta(G)$ leaves.

5. Prove that for every connected graph G, $rad(G) \leq diam(G) \leq 2\, rad(G)$.

6. Let T be a tree with k edges, and let G be a simple n-vertex graph with more than $n(k-1) - \binom{k}{2}$ edges. If $n > k$ then prove that T is a subgraph of G.