MATH 454: Homework #7

Due Wednesday, 3/6, before 11:59pm via a single PDF file uploaded to the Homework#7 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

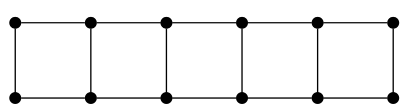
All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

Submit any four out of the five problems stated below.

1. Prove the following:
(a) Let T be a tree. Then each vertex v appears $d_T(v) - 1$ times in the Prufer code of T.
(b) Count the number of n-vertex trees that have $n - 2$ leaves.

2. Let G_n be the graph on $2n$ vertices and $3n - 2$ edges with the structure as shown in G_6 below (two n-vertex paths with matching of n edges between them). Prove that $\tau(G_n) = 4\tau(G_{n-1}) - \tau(G_{n-2})$, for $n \geq 3$.



3. Use Matrix Tree Theorem to prove Cayley's Formula.

4. Compute $\tau(K_{2,m})$, the number of spanning trees of $K_{2,m}$.

5. Prove any one of the following two statements:
(a) If no two edge weights of a connected graph G are equal, then G has a unique minimum spanning tree.
(b) Let e be the unique edge of minimum weight in a weighted connected graph G. Then e must be an edge in every minimum spanning tree of G.
Comment: Just review the definition of minimum spanning tree from Section 2.3; you don't need to know anything else from that section to solve this problem.