# MATH 454: Homework #9

**Due Wednesday, 3/27, before 11:59pm via a single PDF file uploaded to the Homework#9 under Assignments in the Blackboard course page.**

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "HW Discussion and Solution Rules" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during my *Office Hours*, during the *TA office hours*, or through *Email to me*.

**Submit any four (including problem 1 which is compulsory) out of the problems 1-5 stated below. Problem 6 is optional and may be submitted as an extra credit problem.**

**1. (a)** Which of the following relations is true for every $G$ and $H$, where $H$ is a subgraph of $G$? Assume both $G$ and $H$ have minimum degree at least 1.
(i) $\alpha(H) \leq \alpha(G)$; (ii) $\alpha'(H) \leq \alpha'(G)$; (iii) $\beta(H) \leq \beta(G)$; (iv) $\beta'(H) \leq \beta'(G)$.

**(b)** Let $G$ be a graph with $n$ vertices and $m$ edges.
Show that $\alpha(G) \leq m/\delta(G)$ and $\alpha(G) \geq n/(\Delta(G)+1)$.

**2.** Prove, using Konig-Egervary Theorem, that every bipartite graph $G$ has a matching of size at least $|E(G)|/\Delta(G)$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least $k$.

**3.** Prove that $\alpha(G) \leq |V(G)| - |E(G)|/\Delta(G)$ for any non-trivial graph $G$. Conclude that $\alpha(G) \leq |V(G)|/2$ when $G$ is also regular.

**4.** Let $G$ be an $n$-vertex simple graph. If $\delta(G) \geq (n+k-2)/2$ then $G$ is $k$-connected (where $1 \leq k \leq n-1$).

**5.** Solve the Exercise 3.1.28 from the textbook (page 120).

**6.** Prove that a tree $T$ has a perfect matching if and only if $q(T - v) = 1$ for every $v \in V(T)$. Recall that $q(H)$ is the number of odd components in $H$.