

**COURSE INFORMATION: Spring 2025**  
**MATH 554 Modern Methods in Discrete Applied Mathematics**

**Time and Place:** 3:15pm-4:30pm, Monday & Wednesday, at RE 119.

**Instructor:** [Hemanshu Kaul](#).

**Office:** 125C, Rettaliata Engg Center.

**E-mail:** [kaul@iit.edu](mailto:kaul@iit.edu)

**Online Discussion:** [Math 554 at Canvas](#).

**Office Hours:** Monday and Wednesday at 4:30-5:30pm. And by appointment in-person or through Zoom (send email to setup appointment).

Questions through Canvas (above) are strongly encouraged.

**Course Communications:** **Course Webpage**

Check the course webpage regularly for homework assignments, announcements, and a lecture log with notes.

I often send emails with comments regarding HW problems, Exams, etc. Make sure your IIT email account is active and working.

**Textbook:** There is no required textbook. See below for a discussion of various possible supplementary textbooks.

**Grade Break-down:** Homework worth 35%; One mid-term exam worth 25%; Final exam worth 25%; and a project worth 15%. The grading scale will be no more strict than A:85-100, B:73-84, C:60-72.

**Weekly Schedule:** On *Monday and Wednesday* of each week, we will hold lectures and discussion in the classroom. A lecture log along with lecture notes will be available for review each week. Once every one or two weeks, a HW, due either one or two weeks later, will be uploaded to the course webpage.

**Class Attendance and Participation:** You are expected to attend all the lectures, and participate in the class discussions.

**Examinations:** There will be a mid-term exam and a final exam. The exam dates and their precise topics will be announced in class and on the course webpage. Each of the exams might include a take-home component. Make-up exams will be given only in case of a documented emergency.

**Project:** Each student, in consultation with the instructor, will pick a topic, application, or research paper(s) not covered in class. He/She will be expected to write a 10-15 page summary of the topic in their own words and present it in a 20-25 minute lecture in class. The summary should include a short background/ history of results in the area, description of the problem with examples, and overview of the results and the proof techniques used therein. The topic/ paper(s) must be finalized, with the instructor's approval, by **March 10th**. All reports must be submitted by **May 2nd**.

**Prerequisites:** You need to have some familiarity with the topics listed below at basic undergrad level. Don't worry if you don't know each and every topic listed below, in that case you just need to be willing to learn whenever something is needed in the course. I will review topics when needed, and will be available to help you better understand any topics you haven't studied before.

1. Graph Theory: Trees, bipartite graphs, spanning trees, independent set, clique, vertex and edge covers, connectivity, chromatic number, edge chromatic number. Suggested Books - West, *Intro to Graph Theory*, 2nd ed; Diestel, *Graph Theory*, 3rd ed onwards.
2. Combinatorics: Basic Counting (permutations and combinations, various ways of sampling, partitions of integers), pigeonhole principle, Inclusion-Exclusion principle. Suggested Books - any standard undergrad textbook like Brualdi, *Introductory Combinatorics*; Roberts and Tesman, *Applied Combinatorics*; van Lint and Wilson, *A Course in Combinatorics*.
3. Linear Algebra: Vector spaces - definition and lots of examples, linear independence, spanning set, basis and dimension of a vector space, eigenvalues of a matrix. Suggested Books - any standard undergrad textbook.
4. Probability: Random variables (discrete and continuous), expectation, conditional expectation, variance, elementary properties of distributions like Bernoulli, Binomial, discrete uniform, continuous uniform, exponential, Poisson, Normal; discrete Markov chains. Suggested Books - any standard textbook like Stirzaker, *Elementary Probability*; Grimmett and Stirzaker, *Probability and Random Processes*.

**Course Description - Topics and Textbooks:** This graduate-level course in Discrete Mathematics will introduce students in Applied Mathematics, Computer science, Physics, and Engineering, to the use of tools and techniques from various fields of mathematics like Probability, Linear Algebra, Algebra, and Stochastic processes, to existential and algorithmic problems arising in Graph Theory, Combinatorics, and Computer science.

The tools considered would include Probabilistic Methods, Linear Algebra methods, Polynomial method and Combinatorial Nullstellensatz, Entropy, Martingales and large deviation bounds, Markov chain Monte Carlo, etc. These tools will be applied to various fundamental problems like - Graph and Hypergraph coloring, Intersecting families of sets, Ramsey problems, Extremal problems on Graphs and on Set systems (Hypergraphs), Optimization problems on discrete structures, Sampling and counting discrete structures, etc.

There is no one textbook that covers all the topics that I plan to present. So, it will be important that you attend classes regularly and take lecture notes. West, *Combinatorial Mathematics*, recently published, contains many of our topics. Jukna, *Extremal Combinatorics with applications to computer science*, contains some material on probabilistic, linear algebraic, and algebraic methods. Habib, McDiarmid, Ramirez-Alfonsin, and Reed, *Probabilistic methods for algorithmic discrete mathematics*, even though its 20 years old now, has many relevant introductory survey articles.

For Probabilistic Methods, Alon and Spencer, *Probabilistic Method*, 3rd ed. onwards, is the best reference.

For Linear Algebra methods, Babai and Frankl, *Linear Algebra Methods in Combinatorics*, available online, is the standard reference.

For MCMC, Levin, Peres, and Wilmer, *Markov Chains and Mixing Times*, or Haggstrom, *Finite Markov Chains and Algorithmic Applications*, is a good reference.

**Homework Assignment:** A total of 7 homework will be assigned over the semester.

It is your responsibility to check the course webpage for assignments and their due dates. Homework needs to be submitted in class to the instructor on the due date.

HW solutions must be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Pay close attention to the discussion in the sections below.

**HW Discussion Rules:** You are allowed to discuss homework problems **only with your classmates, and me**. However, the solutions should be written by you alone and, if you discussed HW problems with a classmate, you have to **write their name at the top of the HW submission as a collaborator**. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with according to University rules.

**HW Solutions:** When grading your work, I will pay close attention to the following **fundamental aspects of a solution**:

1. Is your proof mathematically correct? Does it start from the correct assumptions and does it conclude in the correct final statement? Is each intermediate statement correct? Is the logical implication in each of your steps correct? Does your proof contain any gaps? Are there any unjustified assumptions? Are there any aspects of the problem that you have not considered?
2. Have you justified the reasons for each of your steps and intermediate statements? Have you clearly explained the thinking and logic underlying your solution? Can the reader follow your justification and explanation? Will the reader be convinced by the detail of your explanation?
3. Does your writing clearly express the mathematical content of your solution? We can only grade based on what you have explicitly written, and not based on your underlying/unexpressed intent. Have you explicitly defined the notation, the variables and the functions you are working with? Have you included appropriate introductory or concluding comments that give context to the problem and your solution?
4. Is your solution readable, particularly, is your handwriting legible and have you used proper indentation and typesetting? Have you corrected any obvious misspellings or incorrect grammar?

**‘Why and How’ of Homework:** In this course, Homework is an opportunity for you to test your understanding of the material covered in class and to explore the topics beyond what we discussed in class. Mathematics cannot be learned by listening or just reading a book - you have to **do it**. Considering the varying pace of learning of students in class and the lack of class time to explore every detail of every concept/Theorem, working through problems in the HW is an easy way for you to make sure that you are keeping up with the class. This is why homework is given a lot of importance in this course - dedicate enough time to it every week.

To improve your mathematical writing quickly, start by writing draft solutions to the homework early. A day or two later after you have had time to forget what you wrote, read it. If it doesn't make sense or convince you, rewrite it. **Writing a solution** requires saying what you mean and meaning what you say. Be intellectually honest. Intellectual dishonesty includes: 1) stating a “reason” without understanding its relevance. 2) Claiming a conclusion when you know you haven't proved it. 3) Giving an example and claiming you have proved the statement for all instances. **Include enough detail in your solutions so that your explanation is convincing to someone who hasn't thought about the problem before**. The proofs should be presented so that your classmates could read them and follow the logic (step-by-step).

Most HW problems will be challenging and not just a simple application of the definitions or theorems. Don't be disheartened if some problems take some time to solve. Such problems help develop your mathematical creativity. Discuss such problems with your classmates, and/or ask me for help, but only after you have given them sufficient thought. Please remember that **homework is NOT meant to be an examination, it is meant to assist in your learning and development**. If you need help with it, don't hesitate to ask.

**Ask for Help:** You are encouraged to ask questions during the *class*, through the *Canvas discussion forums*, during the *Office Hours*, or through *Email to me*. If you are having trouble understanding a homework problem, I will be glad to direct you in the right direction without giving away the solution. The same goes for any concept you have difficulty understanding.

**Don't hesitate to ask for help! I cannot help you if you don't take the initiative.**

### **Accommodations through the Center for Disability Resources:**

Reasonable accommodations will be made for students with documented disabilities. In order to receive accommodations, students must obtain a letter of accommodation from the Center for Disability Resources and email me to make an appointment to speak with me as soon as possible. See the [CDR website](#) for more details.

### **Illinois Tech's Sexual Harassment and Discrimination Information:**

Sexual harassment, sexual misconduct, and gender discrimination by any member of the Illinois Tech community is prohibited. This includes harassment among students, staff, or faculty. Sexual harassment by a faculty member or teaching assistant of a student over whom they have authority or by a supervisor of a member of the faculty or staff is particularly serious. Such conduct may easily create an intimidating, hostile, or offensive environment.

Illinois Tech encourages anyone experiencing sexual harassment or sexual misconduct to speak with the Title IX Office for information on the resolution process and support options.

You can file a complaint [electronically](#), which may be completed anonymously. You may also file a complaint in-person by contacting the Title IX Coordinator, Virginia Foster at 312.567.5725/ [foster@iit.edu](mailto:foster@iit.edu). See [further information here](#).

If you are not ready to file a formal complaint but wish to learn about your rights and options, you may contact Illinois Tech's Confidential Advisor service at 773.907.1062. You can also contact a licensed practitioner in Illinois Tech's Student Health and Wellness Center at 312.567.7550

For a comprehensive list of resources regarding counseling services, medical assistance, legal assistance and visa and immigration services, you can visit [the Title IX Office's website](#).