What is this course *really* about?
aka My aim for this course

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According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic
2. Being able to substitute numbers in 'formulas'/ being able to state or use elementary properties of concepts
3. Given 'formulas'/ elementary properties of a concept, being able to get other 'formulas'/ elementary properties
4. Being able to understand hypotheses and conclusions of theorems
5. Being able to understand the proofs of theorems, step by step
6. Being able to *really* understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems
7. Being able to generalize and extend theorems, and apply them to seemingly unrelated problems
8. Being able to see new relationships, and discover and prove entirely new theorems.

The word 'theorem' is used above in a very general sense - it can also represent algorithms and techniques with a mathematical basis.

Levels 5 and 6 would be considered basic mathematical ability for Math majors. Non-trivial applications of Mathematics would lie in-between levels 6 and 7. While levels 7 and 8 constitute research in Mathematics. A lot of engineering and physics is deep applied mathematics and requires understanding at or beyond levels 6 and 7.

Calculus courses focus on a mixture of 1 and 2. Math 230 (Introduction to Discrete Mathematics) focuses on 3 and 4. Math 332 (Matrices) focuses on 3 and 4 with a bit of 5 . In this course, the focus is more on the upper part of levels 3, 4, 5, and 6.

In the first half of the course, the focus is on 3, 4, and 5 under the context of very elementary properties of integers and solving linear congruences (essentially linear equations modulo an integer whose solutions are required to be integers), and systems of linear congruences. In the second half the focus shifts away from 3 and 4 towards 5 and 6. Along the way we will understand how and when quadratic congruences can be solved (unlike usual quadratic equations, they are hard to solve), properties of prime numbers (the building blocks of integers), algorithmic issues and applications related to these concepts, and famous unsolved problems.

For some of you this will be the first time proofs are featured so prominently in a course, and the first time you study abstract functions which can not be defined using a simple analytic formula. Both these aspects are an important feature of all upper-level math courses. This course aims to help you transition to this new way of thinking and doing mathematics.

I hope this course will help you make progress through these levels of mathematical understanding, and mathematical maturity. I would consider this a successful course, if you gain confidence in your ability to read, understand, and write mathematical arguments (including proofs), especially as compared to the beginning of the semester. And, you have the confidence that you can read, understand, and apply any topic/ technique in Elementary Number Theory whenever you need it.