Proposal for a course:
Convex and discrete geometry

The extent of geometric ideas that a student is exposed to in undergraduate courses is usually limited to topics in coordinate geometry (in 2 and 3 dimensions) and advanced calculus. Moreover, advanced courses in geometry only include differential and algebraic geometry, which focus on developing their abstract tools rather than purely geometric notions. This leaves a big gap in the elementary geometric knowledge of most graduate students, especially in topics from convex and discrete geometry. These topics comprise an active area of research into fundamental geometric questions dating back to antiquity. Their modern applications tie together a wide range of mathematical fields like number theory, combinatorics, optimization, computational geometry, etc.

This gap could be addressed by developing an introductory course in geometry for graduate students. This course should be accessible to all beginning graduate students and include a variety of fundamental topics (some of which are suggested below). Such a course would be of interest not only to students in mathematics but also computer science (computational geometry) and ECE (coding theory, robot motion).

Suggested topics:

0. Review of basic geometric concepts and parameters - like volume, surface area, centroid, etc. for various fundamental bodies, various (affine) transformations, etc. - focusing on high dimensions.

1. Basic convexity theory - including the theorems of Helly, Radon, Carathéodory, and their applications and extensions.

2. Convex bodies - Convex polytopes, approximating convex bodies by convex polytopes and ellipsoids, volumes in high dimensions , hardness of volume approximation, Brunn-Minkowski theory, Borsuk's partition problem.

3. Lattices - Minkowski's Fundamental theorem, Minkowski-Hlawka theorem, geometry of numbers.

4. Tiling, Packing, and Covering - Tiling with regular bodies (for the plane as well as high dimensions), packing and covering with convex bodies/balls (including translation, lattice, and congruent variants), methods of Blichfeldt and Rogers, homothetic covering and illumination problems.

5. Applications - in number theory, extremal combinatorics, coding theory, computational geometry, etc.

References :

Ball - An elementary introduction to modern convex geometry (In Levi, ed., *Flavors of geometry*)
Berg, van Kreveld, Overmars, & Schwarkopf - *Computational Geometry*
Boltyanski, Martini, & Soltan - *Excursions into Combinatorial Geometry*
Conway & Sloan - *Sphere packings, lattices, and groups*
Erdös, Gruber, & Hammer - *Lattice points*
Grunbaum - *Convex polytopes*
Matoušek - *Lectures on discrete geometry*
Pach & Agarwal - *Combinatorial geometry*
Schneider - *Convex bodies: The Brunn-Minkowski theory*
Ziegler - *Lectures on Polytopes*