Math 535 – Optimization I

Course Description from Bulletin: Introduction to both theoretical and algorithmic aspects of linear optimization: geometry of linear programs, simplex method, anticycling, duality theory and dual simplex method, sensitivity analysis, large scale optimization via Dantzig-Wolfe decomposition and Benders decomposition, interior point methods, network flow problems, integer programming. Credit may not be granted for both MATH 435 and MATH 535. (3-0-3)

Enrollment: Graduate Elective.

Textbook(s): Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, 1997.

Other required material: None

Prerequisites: Undergraduate course in elementary linear algebra (such as MATH 332), or consent of instructor.

Objectives:

1. Students will develop the ability to formulate optimization problems, recognize the main classes of problems that are practically solvable, apply available solution methods, and understand the qualitative properties of solutions.
2. Students will develop insight into the geometric basis of linear programs, and the interplay between geometry and linear algebra in their solution methods.
3. Students will understand and apply theorems and algorithms from simplex methods, duality theory, sensitivity analysis, decomposition methods, and select topics from interior point methods, network flows, and integer programming.
4. Students will practice their knowledge through problems that emphasize analytic properties or computational aspects, including the possible use of linear programming solver software.
5. Students will do a project with presentations on a topic approved by the instructor. Presentation topics can include (computational) applications of the course material to student's own research area, and expository talks (with proofs) on material not covered in class.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:	Hours
1. Introduction	2
LP: formulations and examples	
Piecewise linear convex objective functions	
Graphical representation and solution	

2. Geometry of Linear Programs
   5
   - Polyhedra and convex sets
   - Extreme points, vertices, and basic feasible solutions
   - Degeneracy of basic solution
   - Existence and optimality of extreme points
3. Simplex Method 9
   - Optimality conditions
   - Simplex method
   - Revised simplex method and full tableau implementation
   - Anticycling: Bland's rule
   - Initial basic feasible solution
   - Computational efficiency of the simplex method
4. Duality Theory and Sensitivity analysis 7
   - Dual linear program
   - Duality Theorems and Complementary Slackness
   - Dual Simplex method
   - Farkas' Lemma and its application to duality theorem
   - Sensitivity analysis and Parametric programming
5. Large Scale Optimization 5
   - Delayed column generation and Dantzig-Wolfe decomposition
   - Cutting plane methods and Benders decomposition
6. Optional Topics (selected based on class composition and background) 11
   - Interior Point Methods
     - The von Neumann algorithm
     - The affine scaling algorithm
     - The primal path following algorithm
   - Network Flow Problems
     - The minimum cost flow problem and the Network simplex algorithm
     - The maximum flow problem and the Ford-Fulkerson algorithm
     - The assignment problem and the Auction algorithm
   - Integer Programming
     - Gomory Cuts and Cutting plane algorithms
     - Branch and bound
     - Dynamic programming
     - IP duality and Lagrangian Relaxation

7. Exams and Overflow 3

Assessment:

Homework	25-50%
Project	10-20%
Quizzes/Tests	30-50%
Final Exam	25-40%

Syllabus prepared by: Hemanshu Kaul
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