

MATH 553 - Discrete Applied Mathematics I

Course Description from Bulletin: A graduate-level introduction to modern graph theory through existential and algorithmic problems, and the corresponding structural and extremal results from matchings, connectivity, planarity, coloring, Turán-type problems, and Ramsey theory. Proof techniques based on induction, extremal choices, and probabilistic methods will be emphasized with a view towards building an expertise in working in discrete applied mathematics. (3-0-3)

Enrollment: Graduate Core Course

Textbook(s): Diestel, *Graph Theory, 4th ed.*, Springer-Verlag, supplemented with handouts

Other required material:

Prerequisites: MATH 454 or instructor's consent.

Objectives:

1. Students will be proficient in basic definitions and properties of graphs.
2. Students will be proficient in different proof techniques and problems with a view towards building expertise in working in discrete mathematics.
3. Students will be introduced to the different perspectives in modern graph theory through the study of existential and algorithmic questions, and through structural and extremal results.
4. Students will learn fundamental results in structural graph theory, including topics such as matchings, connectivity, and planarity.
5. Students will learn fundamental questions and techniques in extremal graph theory, including topics such as graph coloring and its variants, Turán-type problems, and Ramsey theory.
6. Students will be introduced to basic techniques from probabilistic methods -- existential arguments and their modifications and Lovasz local lemma -- as applied to graph theory.

Lecture schedule: Three 50-minute (or two 75-minute) lectures per week

Course Outline:

1. Graph theory fundamentals
 - a. Basic definitions, isomorphism, matrices
 - b. Paths, walks, trails, and extremal choice
 - c. Bipartite graphs and odd cycles

Hours

6

- d. Eulerian circuits
- e. Trees, spanning trees, and forests
- 2. Structural graph theory 8
 - a. Matchings and Coverings
 - i. Bipartite graphs: Hall's Theorem and König-Egerváry Theorem
 - ii. General graphs: Tutte's Theorem
 - b. Connectivity and Menger's Theorem (with proof)
 - c. Planarity and Crossing number
- 3. Coloring 9
 - a. Basics of vertex and edge coloring
 - b. List coloring and other variants
 - c. Probabilistic and other methods in graph coloring
- 4. Extremal problems 8
 - a. Turán's Theorem, Erdos-Stone
 - b. Forbidden subgraph problems
 - c. Zarankiewicz's problem
 - d. Probabilistic methods in extremal problems
- 5. Ramsey theory 8
 - a. Ramsey's theorem for graphs
 - b. Graph Ramsey theory
 - c. Probabilistic methods in Ramsey theory

Assessment:	Homework	10-50%
	Quizzes/Exams	20-50%
	Final Exam	30-50%

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