Math 431, Homework assigned on 4/22/2013, Pelsmajer

(1) For each of these codes, how many errors can be detected?

. . . . . Corrected?

(a) $\{0000, 1100, 1010, 1001, 0110, 0101, 0011, 1111\}$ in $GF(2)^4$

(all elements with <u>even</u> weight)

(b) $\{10000, 01010, 00001\}$ in $GF(2)^5$

(c) $\{000000, 101010, 010101\}$ in $GF(2)^6$

(d) $\{000000, 121000, 212000, 000112, 000221\}$ in $GF(3)^6$

(2) Prove the "triangle inequality" for Hamming distance:

$$d(x,y) \le d(x,z) + d(z,y) \quad \text{for any } x,y,z \in V = GF(q)^n$$

(3) Which of these are cyclic?

$\{000, 100, 010\}$

$\{000, 100, 010, 001\}$

$\{000, 111\}$

$\{0000, 1010, 0101, 1111\}$

(4) Write down the code words of the cyclic code corresponding to $\langle 1+x+x^2 \rangle$ in $F[x]/\langle x^3-1 \rangle$ with $F = GF(2)$.

(5) Write down the cyclic code corresponding to $\langle 1-x \rangle$ in $F[x]/\langle x^3-1 \rangle$ with $F = GF(4)$.

(6) Find all ~~the~~ irreducible factors of x^5-1 over $GF(2)$, and then determine all (four) (dull) cyclic codes of length 5.