Math 453, Spring 2008, Instructor: Pelsmajer
Assigned on Feb 3.

As in class, we use "Part(k, n)" to denote the number of *partitions of k into n parts*, which can be defined as the number of ways to divide k identical objects (like ping-pong balls) into n unlabelled bags, such that no bag is empty. Alternatively, this is the number of multisets of k positive integers that sum to n.

(A) Generate enough of the "Partition Triangle" (analogous to Pascal's triangle) so that you find the number of ways to add up 5 positive integers such that the sum equals 12 (where the order of the 5 integers doesn't matter).

(You should use the recurrence from class, but first you'll have to find Part(k, n) for certain k, n; these values should be easy to justify, and you should justify these explicitly.)

(B) Looking at the row/column-switching map on Ferrars diagrams, finish the following:

The number of partitions of k into *at most n parts* is equal to the number of partitions of k into . . .

(C) Show that $\text{Part}(k, n) \geq \binom{k-1}{n-1}/n!$.

Hints: Show that $\binom{k-1}{n-1}/n!$ sort of counts the same thing as Part(k, n), except that when we divide by $n!$ we are doing too much. (After you try that, see what that thought process looks like for specific examples; say, $k = 6$ and $n = 3$, etc.)

Alternatively, show that $n!\,\text{Part}(k, n)$ and $\binom{k-1}{n-1}$ sort of count the same thing, except... etc., etc.

From 2.8, do #5, 7, 8.

From 2.18, Read as necessary (probably two theorems) and do #9.

From 2.19, do #1a, 2a, 7, 8, 9

For those with less background in graph theory, you might want to review some basic terminology: vertex/vertices, edge, complete graph, path & walk, cycle/circuit, multiple edges & loops.

Any book and/or wikipedia will do. Some of these definitions are not consistent among different sources; we'll deal with that as necessary.

Due on Tuesday, Feb 5, along with the homeowork assigned earlier.