As in class, we use “Part(k, n)” to denote the number of partitions of k into n parts, which can be defined as the number of ways to divide k identical objects (like ping-pong balls) into n unlabelled bags, such that no bag is empty. Alternatively, this is the number of multisets of k positive integers that sum to n.

(A) Generate enough of the “Partition Triangle” (analogous to Pascal’s triangle) so that you find the number of ways to add up 5 positive integers such that the sum equals 12 (where the order of the 5 integers doesn’t matter).

(You should use the recurrence from class, but first you’ll have to find Part(k, n) for certain k, n; these values should be easy to justify, and you should justify these explicitly.)

(B) Looking at the row/column-switching map on Ferrars diagrams, finish the following:

The number of partitions of k into at most n parts is equal to the number of partitions of k into . . .

(C) Show that \( \text{Part}(k, n) \geq \frac{(k-1)}{(n-1)!}/n! \).

Hints: Show that \( \frac{(k-1)}{(n-1)!}/n! \) sort of counts the same thing as Part(k, n), except that when we divide by n! we are doing too much. (After you try that, see what that thought process looks like for specific examples; say, \( k = 6 \) and \( n = 3 \), etc.)

Alternatively, show that \( n! \text{Part}(k, n) \) and \( \frac{(k-1)}{(n-1)!} \) sort of count the same thing, except... etc., etc.

From 2.8, do #5, 7, 8.
From 2.18, Read as necessary (probably two theorems) and do #9.
From 2.19, do #1a, 2a, 7, 8, 9

For those with less background in graph theory, you might want to review some basic terminology: vertex/vertices, edge, complete graph, path & walk, cycle/circuit, multiple edges & loops.

Any book and/or wikipedia will do. Some of these definitions are not consistent among different sources; we’ll deal with that as necessary.

Due on Tuesday, Feb 5, along with the homework assigned earlier.