Math 453, Spring 2008, Instructor: Pelsmajer

Assigned on Feb 5, revised problem (B) on Feb 6.

(A) Let V be a set of n vertices, and assign color red, blue, or green to each subset of size 2. (Or, equivalently, color the edges of K_n.) Say that $U \subset V$ *is red* if every 2-subset of U is colored red. (Similar for blue and green.) Also, a *k-set* is a set of k vertices.

Prove that for each p, q, r there is a number $R(p, q, r)$ such that:
if $n \geq R(p, q, r)$, then there must be a red p-set, or a blue q-set, or a green r-set.

For problem (B), choose option 1 xor option 2. You cannot do the (easy) bonus if you choose option 1.

(B, option 1) Using the recurrence for $R(p, q)$ from class, prove that $R(p, q) \leq 2^{p+q}$.

(B, option 2) Using the recurrence for $R(p, q)$ from class, prove that $R(p, q) \leq \binom{p+q-2}{p-1}$.
(B, option 2 bonus) Then use Stirling's approximation (page 26) on $\binom{p+q-2}{p-1}$ to find an asymptotic upper bound for $R(p, p)$.

(C/Bonus) In class I mentioned that you can't have a Ramsey-type theorem that guarantees to have 3 points in a line; i.e., simply having a lot of points in the plane will never force you to have 3 on a line.

(i) What about a rectangle? I.e., does there exists a number N such that if you have at least N points in the plane, you're guaranteed to have 4 points that form the corners of a rectangle?

(ii) What about n points *not* in convex position? I.e., does there exist a number $Q(n)$ such that whenever you have at least $Q(n)$ points in the plane, you're guaranteed to have n points that are not in convex position?

(D) We already know that $\text{Part}(k, n) = \sum_{j \leq n} \text{Part}(k - n, j)$, or written differently,

$$\text{Part}(k, n) = \sum_{j=0}^{n} \text{Part}(k - n, j).$$

Explain why the "$j = 0$" is correct
(i) when $k = n$,
(ii) when $k > n$.

(Case (i) shows why the sum must begin at $j = 0$ instead of $j = 1$.)