Section 2.9, do #5, 6, 8
Section 2.10, do #16, 17
Section 2.11, do #7a, 10, 11
Section 2.14, Read all, and do #2, 6, 7, 16abce. (For problem #8, see discussion below.)
And 4 more problems, and a bonus problem:
From Section 2.14, the answer to #5 is:

\[(x + y + z)^n = \sum_{i,j,k : i+j+k=n} \binom{n}{i,j,k} x^i y^j z^k.\]

The way you say the right-hand side is, “The sum of \(x^i y^j z^k\), where the sum is taken over all \(i, j, k\) such that \(i + j + k = n\).”

*First*, expand the right-hand side for \(n = 2\) (to verify that you understand what is meant, and that it really equals the left-hand side). *Then*, give a proof of this formula, similar to the proof I presented at the end of class (also on p71).

Finally, give combinatorial/bijective proofs for each of the following:

\[
\binom{k}{\ell} \binom{n}{k} = \binom{n}{\ell} \binom{n - \ell}{k - \ell}
\]

\[\sum_k \binom{n}{k} = 2^n\]

\[\sum_k \binom{m}{k} \binom{n}{r - k} = \binom{m + n}{r}\]

and, as an optional bonus problem:

\[\sum_{k=0}^n \binom{k}{r} = \binom{n + 1}{r + 1}\]

A typical combinatorial/bijective proof: Think of something that is counted by the left-hand side, and then explain why the right-hand side is another way of counting the same thing (or, as in our “gathering” examples from class, the right-hand side is a way of counting something else which is in one-to-one correspondence with the first “something”).

*Or* start on the right, and explain why the left counts the same thing.

(And do everything assigned on Tuesday, Jan 22.)

Due on Tuesday, Jan 29.