Math 453, Spring 2008, Instructor: Pelsmajer
Assigned on Jan 25, corrected on Jan 29 after class.

Section 2.9, do #5, 6, 8

Section 2.10, do #16, 17

Section 2.11, do #7a, 10, 11

Section 2.14, Read all, and do #2, 6, 7, 16abce. (For problem #8, see discussion below.)

And 4 more problems, and a bonus problem:
From Section 2.14, the answer to #5 is:

$$(x+y+z)^n = \sum_{i,j,k\,:\,i+j+k=n} \binom{n}{i,j,k} x^i y^j z^k.$$

The way you say the right-hand side is, "The sum of $x^i y^j z^k$, where the sum is taken *over all* i,j,k *such that* $i+j+k=n$".

First, expand the right-hand side for $n=2$ (to verify that you understand what is meant, and that it really equals the left-hand side). *Then, give a proof* of this formula, similar to the proof I presented at the end of class (also on p71).

Finally, give combinatorial/bijective proofs for each of the following:

$$\binom{k}{\ell}\binom{n}{k} = \binom{n}{\ell}\binom{n-\ell}{k-\ell}$$

$$\sum_k \binom{n}{k} = 2^n$$

$$\sum_k \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$

and, as an optional bonus problem:

$$\sum_{k=0}^{n} \binom{k}{r} = \binom{n+1}{r+1}$$

A typical combinatorial/bijective proof: Think of something that is counted by the left-hand side, and then explain why the right-hand side is another way of counting the same thing (or, as in our "gathering" examples from class, the right-hand side is a way of counting something else which is in one-to-one correspondence with the first "something").

Or start on the right, and explain why the left counts the same thing.

(And do everything assigned on Tuesday, Jan 22.)

Due on Tuesday, Jan 29.