Math 453, Spring 2008, Instructor: Pelsmajer
Assigned on Jan 25.

Read p26.

From 2.10, read examples #2.26-2.30, and do #7, 13, 22ac, 23, 24, 25. (We sketched two of these in class but I want you to write up the proofs clearly, which should solidify/confirm your understanding.) (For #13, I'd phrase it "we only care about which passengers are together, not which floor they exit on".)

(A) Count the number of ways to choose subsets $X_0, X_1, \ldots, X_n$ of $[n]$ such that $X_0 \subset X_1 \subset \cdots \subset X_n$.

(B) Count the number of ways to choose subsets $X_0, X_1, \ldots, X_n$ of $[n]$ such that $X_0 \subseteq X_1 \subseteq \cdots \subseteq X_n$.

(C) Count the lists of m 1's and n 0's that have exactly k runs of 1's. (A *run* is a maximal constant substring.)

(D) Count the number of ways to group $2n$ people into pairs. (The order within each pair does not matter, and the order of the pairs doesn't matter, either.)

(E) Prove

$$\sum_k \binom{k}{\ell}\binom{n}{k} = \binom{n}{\ell}2^{n-\ell}.$$

(F) Prove $\sum_i \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}$.

For each counting problem, you should provide a clear explanation of why it works; i.e., a proof of correctness.

More will be assigned Thursday.
Due on Tuesday, Feb 5.