PRINT Last/Family name:______________________ First/Given name:_________________________

Signature:___________________________________ Student ID:_______________________________

Exam 2

Time limit: 75 minutes (strict), by countdown timer on screen if available.

Instructions. You **MUST** show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions.

- All computers, cell phones, notes, books, hats, etc. must be put away. Cell phones must be silenced, not just on vibrate.
- No calculators or aids of any kind other than a pen or pencil. Do not use your own scratch paper; ask for some.
- By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the Code of Academic Honesty.
- Do not talk before time is expired, even if you finish early.

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Use the back two pages and the area below for **additional workspace**.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x \, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \ge f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \ge f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \ge f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\,dx \le s - s_n \le \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

$\text{Vol} = \int_a^b 2\pi \left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = y \iff f^{-1}(y) = x$

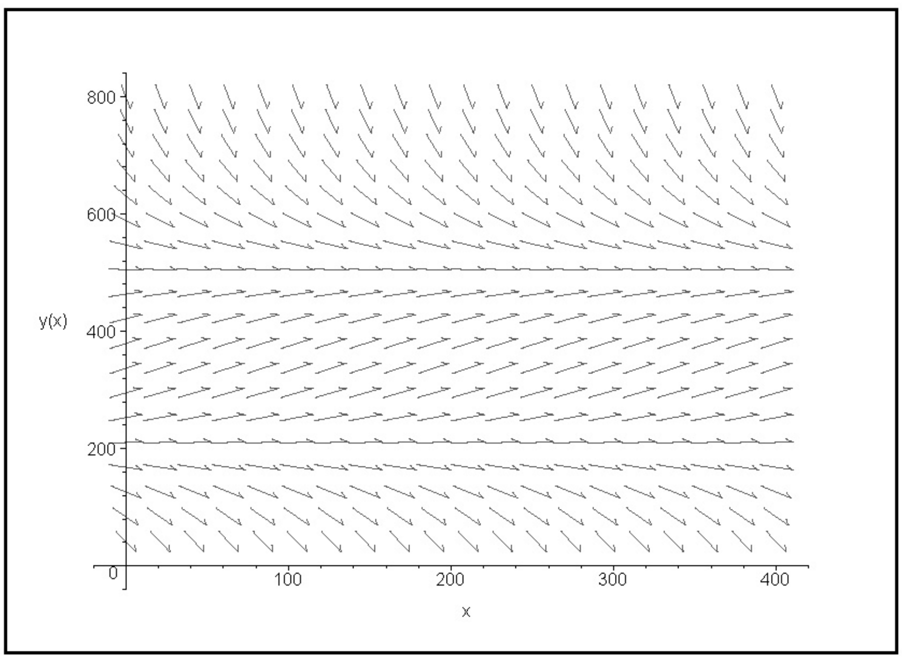
1. Write out the form of the partial fraction decomposition of $\dfrac{x^2-13x-14}{(x-1)(x+3)^2\left(x^2+1\right)^2}$, but **do not** solve for the various coefficients.

2. According to the error bound formulas for the Midpoint and Trapezoid rules for approximate integration, how many subintervals n are required to approximate $\int_0^1 6\cos(2t)dt$ as follows:

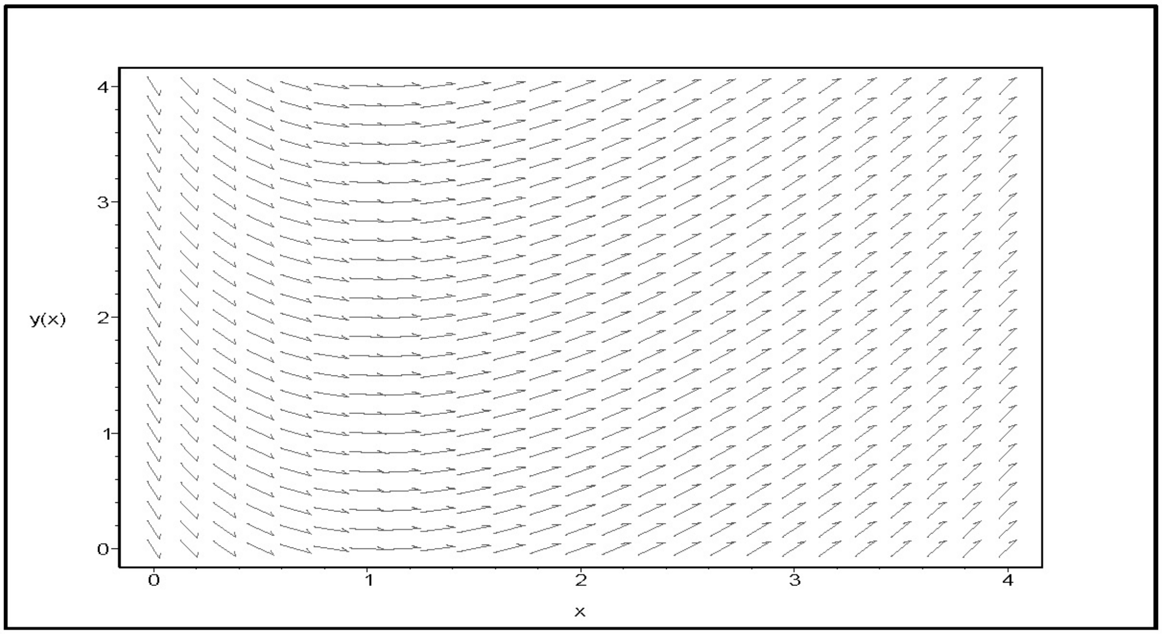
   a) To within 4×10^{-4}, for the Midpoint Rule?
   b) To within 2×10^{-4}, for the Trapezoid Rule?

3. Determine whether $\int_1^\infty \frac{dx}{x^{1/2} - \frac{1}{2}x^{1/4}}$ converges by comparing it to $\int_1^\infty \frac{dx}{x^p}$ for an appropriate value of p. (This requires computing the integral for your value of p.)

4. Sketch four particular solutions to the differential equation whose slope field is below: **(a)** 2 equilibrium solutions, **(b)** 1 increasing solution, and **(c)** 1 decreasing solution.



5. The slope field of a differential equation is represented below.
   a) Trace the particular solution for the initial condition $y(0)=3$.
   b) Use Euler's method with stepsize $\Delta x = 1$ to trace an approximation to the particular solution for the same initial condition $y(0)=3$.
   c) Make a general statement about how Euler's method will deviate from the corresponding exact particular solution given the concavity of that particular solution.



6. Find the solution of the differential equation that satisfies the given initial condition.

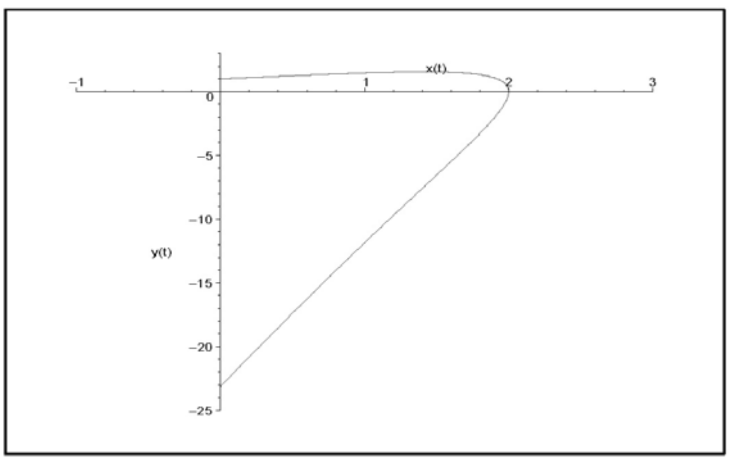
$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}, \quad u(0) = -6.$$

7. Solve the differential equation $y' = 8x - y$.

8. The plot below is generated by a particle following the parametric equations

$$x(t) = 2\sin(t)$$
$$y(t) = e^t \cos(t)$$
$$0 \le t \le \pi$$

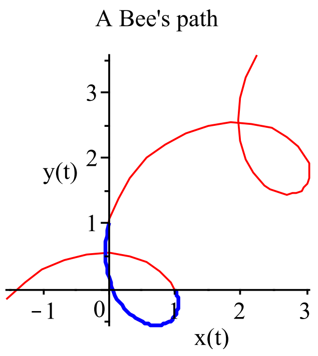
Determine **exactly** the *x*-coordinate $x(t)$ of the rightmost position of the particle, and the value of t for which this occurs.



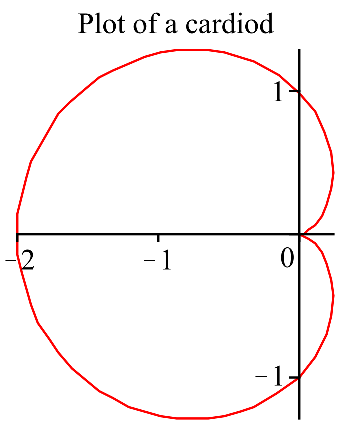
9. A bee buzzes along a path with parametric description

$$x(t) = \frac{t}{2} + \cos\left(\frac{\pi t}{2}\right),$$
$$y(t) = \frac{t}{2} - \sin\left(\frac{\pi t}{2}\right).$$

Set up (**but do not integrate**) an integral representing the arc length of the portion of the path starting at $(x_1, y_1) = (1, 0)$ and ending at $(x_2, y_2) = (0, 1)$. The integrand should be a function of t which contains no derivatives.



10. A cardiod is given in polar form by the function $r(\theta) = 1 - \cos(\theta)$ for $0 \le \theta \le 2\pi$. Find the **horizontal** tangents to the plot of the cardiod, carefully describing the behavior at the cusp (at the pole). (Hint: It may be helpful to convert expressions to either all cosines or all sines.)



[WORKSPACE]