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1. Compute the inverse function of the function $f(x)=\dfrac{4x-1}{2x+3}$.

$$y=\frac{4x-1}{2x+3}$$

$$\boxed{f^{-1}(x)=\frac{3x+1}{-2x+4}}$$

2 $2xy+3y=4x-1$

1 $3y+1=4x-2xy$

2 $3y+1=x(4-2y)$

1 $\dfrac{3y+1}{4-2y}=x$

Version B: $y=\dfrac{2x-5}{4x+3}$

2 $4xy+3y=2x-5$

1 $3y+5=2x-4xy$

2 $3y+5=x(2-4y)$

1 $\dfrac{3y+5}{2-4y}=x$ $\boxed{f^{-1}(x)=\dfrac{3x+5}{2-4x}}$

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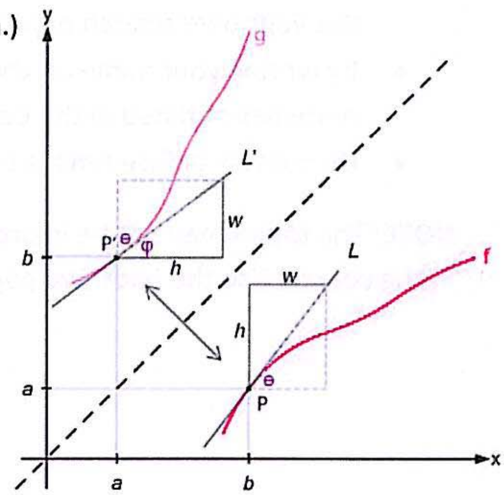
2. Compute $(f^{-1})'(4)$ (the derivative of the inverse function of *f*, evaluated at 4), given that $f(x)=2x^3+3x^2+7x+4$. (The figure hints at the formula.)



2 $(f^{-1})'(4)=\dfrac{1}{f'(f^{-1}(4))}$

1 $f(0)=4 \Leftrightarrow f^{-1}(4)=0$

1 $f'(x)=6x^2+6x+7$

1 $f'(0)=7$

1 $(f^{-1})'(4)=\dfrac{1}{f'(0)}=\boxed{\dfrac{1}{7}}$

Version B: $\dfrac{1}{5}$

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3. Compute the limit: $\lim\limits_{x\to 6^+}\ln(x^2-36)$

$= \cancel{\ln\left(\lim\limits_{x\to 6^+}(x^2-36)\right)}$

3 change of variable:

$u=x^2-36$

as $x\to 6^+$, $u\to 0^+$

3

$=\lim\limits_{u\to 0^+}\ln u=-\infty$

Version B: $\lim\limits_{x\to 4^+}\ln(x^2-16)$

change of variable $u=x^2-16$ 3

as $x\to 4^+$, $u\to 0^+$

$\boxed{\lim\limits_{u\to 0^+}\ln u=-\infty}$ 3

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4. Find an equation of the tangent line to the following curve at the point $(0,1)$.

$$f(x)=e^{3x}\cos(\pi\cdot x)$$

4 $f'(x)=3e^{3x}\cos(\pi x)+e^{3x}(-\pi\sin\pi x)$

1 $f'(0)=3+(-\pi\sin 0)=3$

1 $y-1=3(x-0)$

1 $\boxed{y=3x+1}$

5. Version B $f(x)=(\sqrt{x})^x$

$y=(x^{1/2})^x=x^{\frac{1}{2}x}$

$\ln y=\ln x^{\frac{1}{2}x}=\frac{1}{2}x\ln x$

$\frac{y'}{y}=\frac{1}{2}\ln x+\frac{1}{2}x\left(\frac{1}{x}\right)$

$=\frac{1}{2}\ln x+\frac{1}{2}$

$\boxed{y'=(\sqrt{x})^x\left(\frac{1}{2}\ln x+\frac{1}{2}\right)}$

6

5. Differentiate the function $f(x)=(x)^{\sqrt{x}}$.

2 $\ln y=\ln x^{\sqrt{x}}=\sqrt{x}\ln x$

2 $\frac{y'}{y}=x^{\frac{1}{2}}\frac{1}{x}+\frac{1}{2}x^{-\frac{1}{2}}\ln x$

1 $=\frac{1}{\sqrt{x}}+\frac{1}{2}\frac{\ln x}{\sqrt{x}}$

1 $\boxed{y'=x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+\frac{1}{2}\frac{\ln x}{\sqrt{x}}\right)}$

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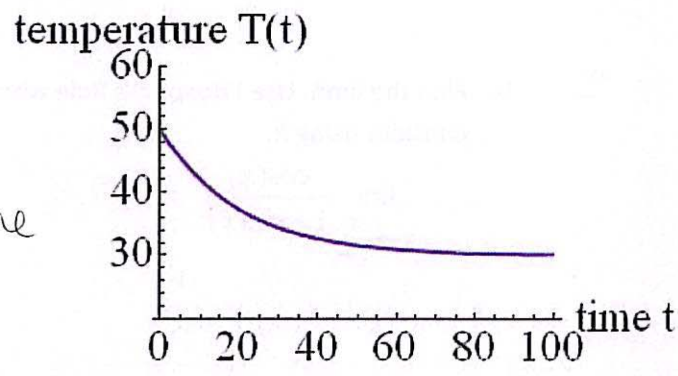
6. The following plot graphs the temperature of an object placed at time zero in an environment with a different temperature.
   2 a) Initially, is the object colder or warmer than its environment?
   2 b) What is the approximate ambient temperature of the environment?
   2 c) Is the exponential rate constant associated with the graph positive or negative, and how can you tell?



Version B	
colder	(a) warmer
50	(b) 30
negative (same reason)	(c) negative because there appears to be a horizontal asymptote
	—or—
rate of increase is decreasing	excess/deficit heat decays.
	rate of decrease is decreasing

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7. An initial amount of \$1000 is invested at the interest rate of 7% compounded continuously. (Decimal approximations are **not** required to answer this question.)

4 a) How long will it take the investment to double?

3 b) What is the equivalent annual interest rate, corresponding to simple interest applied once annually?

2 $A(t) = 1000\, e^{.07t}$

1 $A(t_2) = 2000 = 1000\, e^{.07 t_2}$

$2 = e^{.07 t_2}$

$\ln 2 = .07\, t_2$

doubling time 1 $\boxed{t_2 = \frac{\ln 2}{.07}}$

set $A(1) = 1000(1+s)$

2 $1000\, e^{.07} = 1000(1+s)$

$e^{.07} = 1+s$

~~.07 =~~

1 $\boxed{s = e^{.07} - 1}$

6

8. Find the following limit, if it exists: $\lim_{x\to\infty} \arctan\left(e^{x}\right)$

~~Σare~~ let $u = e^x$.

3 as $x \to \infty$, $u \to \infty$.

so, original $= \lim_{u\to\infty} \arctan(u) = \boxed{\frac{\pi}{2}}$ 3

Version B $\lim_{x\to 0^+} \arctan^{-1}(\ln x)$

$= \lim_{u\to -\infty} \arctan(u)$

$= \boxed{-\frac{\pi}{2}}$

6

9. Compute the exact value of $\arcsin\left(\sin\left(8\pi/3\right)\right)$.

$\sin\left(\frac{8\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. 2

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$ since $-\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}$.

ans. 2 range: 2

7

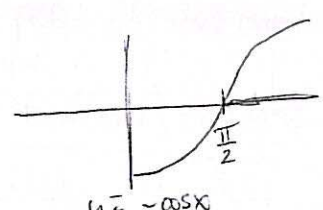
10. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\frac{\pi}{2}^+} \frac{\cos(x)}{1-\sin(x)}$$

$\lim_{x\to\frac{\pi}{2}^+} \cos x = 0$ 1 $\lim_{x\to\frac{\pi}{2}^+} 1 - \sin x = 0$ 1

IF type 0/0. 1

L'H $= \dfrac{\lim_{x\to\frac{\pi}{2}^+} -\sin x}{\lim_{x\to\frac{\pi}{2}^+} -\cos x} = \dfrac{-1}{0} = -\infty$ 1 1 1 1



$y = -\cos x$

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6 11. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} \sqrt{x^2+5x}-x$$

$\sqrt{x^2+5x} - x = \sqrt{x^2(1+5/x)} - x(1)$

For $x\to\infty$: $\stackrel{2}{=} x\left(\sqrt{1+5/x}-1\right)$

$\lim_{x\to\infty} x = \infty$

1 F 1

$\lim_{x\to\infty} \sqrt{1+5/x}-1 = 0$, type $\infty\cdot 0$

$\lim_{x\to\infty} \frac{\sqrt{1+\frac{5}{x}}-1}{1/x}$ L'H $\frac{\lim_{x\to\infty} \frac{1}{2}\left(1+\frac{5}{x}\right)^{-1/2}\left(\frac{-5}{x^2}\right)}{\lim_{x\to\infty} \frac{-1}{x^2}}$ 1 1 $= \lim_{x\to\infty} \frac{5}{2}\left(1+\frac{5}{x}\right)^{-1/2} = \boxed{\frac{5}{2}}$ 1

6 12. Evaluate the integral $\int 2x^2\cos(3x)\,dx = \frac{2}{3}x^2\sin(3x) + \frac{4}{9}x\cos(3x) - \frac{4}{27}\sin(3x) + C$ 1

setup/fmla: 2

$2x^2$	$\cos(3x)$
$4x$	$\frac{1}{3}\sin(3x)$ 1
4	$-\frac{1}{9}\cos(3x)$ 1
	$-\frac{1}{27}\sin(3x)$ 1

(−)

Version B

$3x^2$	$\sin(2x)$
$6x$	$-\frac{1}{2}\cos(2x)$
6	$-\frac{1}{4}\sin(2x)$
	$\frac{1}{8}\cos(2x)$

$-\frac{3}{2}x^2\cos(2x) + \frac{3}{2}x\sin(2x) + \frac{3}{4}\cos(2x) + C$

7 13. Evaluate the integral $\int_{25}^{36} \frac{\ln x}{\sqrt{x}}\,dx = (\ln x)2\sqrt{x}\Big|_{25}^{36} - \int_{25}^{36} \frac{2}{\sqrt{x}}\,dx$

method/fmla: 2

$u = \ln x$ 1 $\quad dv = \frac{1}{\sqrt{x}}\,dx$

$du = \frac{dx}{x}$ 1 $\quad v = 2x^{\frac{1}{2}}$ 1 $\quad = (\ln x)2\sqrt{x}\Big|_{25}^{36} - 4\sqrt{x}\Big|_{25}^{36}$ 1

$= 12\ln 36 - 10\ln 25 - 24 + 20$

$= \boxed{12\ln 36 - 10\ln 25 - 4}$ 1

Version B

$(\ln y)2\sqrt{y} - 4\sqrt{y}\Big|_{16}^{25}$

$= ((\ln 25)\cdot 10 - 20) - ((\ln 16)8 - 16)$

$= \boxed{10\ln 25 - 8\ln 16 - 4}$

19

6

14. Determine the value of the following definite integral, with justification: $\frac{1}{\pi}\int_{-\pi}^{\pi}\sin(x)\cos(2x)\,dx$

$= 0.$

sin(x) is an odd function 1

cos(2x) is an even function. 1

odd fxn · even func = odd function. 1

An odd function integrated on limits symmetric about zero ~~is~~ yields 0. 3

6

15. Evaluate the integral $\int 5\tan^3(x)\sec^4(x)\,dx$

$= \int 5\tan^3(\cancel{x})(\tan^2\cancel{x}+1)\sec^2\cancel{x}\,dt = \frac{5\tan^6 x}{6} + \frac{5\tan^4 x}{4} + C$ 1

$u = \tan\cancel{x}$

$du = \sec^2\cancel{x}\,d\cancel{x}$ 3

$= \int 5u^3(u^2+1)\,du$ 1

$= \frac{5u^6}{6} + \frac{5u^4}{4} + C$ 1

Version B

$\int 5\tan^3 x\sec^5 x\,dx$

$= \int 5\tan^2 x\sec^{\cancel{5}4} x(\sec x\tan x\,dx)$

$= \int 5\tan^2 x\sec^4 x(\sec x\tan x)dx$

$= \int 5(\sec^2 x-1)\sec^4 x(\sec x\tan x)dx$

$u = \sec x \quad du = \sec x\tan x\,dx$

$\int 5(u^2-1)u^4\,du = 5u^7/7 - 5u^5/5 + C$

$= \boxed{\frac{5}{7}\sec^7 x - \sec^5 x + C}$

6

16. Evaluate the integral: $\int_0^1 5x^3\sqrt{1-x^2}\,dx$

1 $x = \sin t \quad (-\frac{\pi}{2} \le t \le \frac{\pi}{2})$

$dx = \cos t\,dt$

1 $\sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \cos t$

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = \frac{\pi}{2}.$

$= 1\int_0^{\pi/2} 5\sin^3 t\cos t\cos t\,dt$

1 $= \int_0^{\pi/2} 5\sin t(1-\cos^2 t)\cos^2 t\,dt$

$w = \cos t \quad dw = -\sin t\,dt$

1 $= \int_{t=0}^{t=\frac{\pi}{2}} -5(1-w^2)w^2\,dw$

$= -\frac{5w^3}{3} + w^5 \Big|_{t=0}^{t=\pi/2}$

$= -\frac{5\cos^3 t}{3} + \frac{\cos^5 t}{1}\Big|_0^{\pi/2} = +\frac{5}{3} + (-1) = \boxed{+\frac{2}{3}}$ 1

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