

PRINT Last name: Key B First name: _____
 Signature: _____ Student ID: _____

Instructions: Show work for full credit. No notes, calculators, hats, cell phones, or aids of any kind. 15-minute time limit. By signing your name you agree that all work is your own.

Clearly circle or otherwise indicate your answer in order to get credit.

Questions 1-5 are 1pt each with no partial credit.

Questions 6-7 are 2pts each, and all work must be shown.

1. Suppose r and w are the propositions

b

r : "it is rainy"

w : "it is windy"

Express in English a compound proposition that is logically equivalent to $w \rightarrow \neg r$.

(a) Being windy is sufficient for it to be rainy.

☒ (b) If it is rainy, then it is not windy.

(c) It is windy, but not rainy.

(d) Whenever it is not windy, it is rainy.

e

2. Which of the following is the negation of the statement "I drive to work if and only if it is rainy?"

(a) I drive to work if it is not rainy.

(b) I do not drive to work if it is not rainy.

(c) I do not drive to work if and only if it is not rainy.

(d) If I drive to work, then it is not rainy.

☒ (e) I drive to work if and only if it is not rainy.

d

3. If x , y , and z are real numbers, $x < y < z$ means that x is less than y and y is less than z . Which of the following is the negation of $x < y < z$?

(a) $x > y > z$

(b) $x \geq y \geq z$

(c) $x > z$

☒ (d) $x \geq y$ or $y \geq z$

b

4. Suppose m and a represent the propositions:

m : "you are a member of the committee"

a : "you attend the meeting."

Express in symbols the compound proposition

"To be a member of the committee it is necessary that you attend the meeting."

(a) $\neg m \rightarrow a$

☒ (b) $m \rightarrow a$

(c) $a \rightarrow \neg m$

(d) $a \rightarrow m$

Q 5. Which of the following propositions is correct?

- (a) The converse of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
 (b) The converse of the implication $p \rightarrow q$ is logically equivalent to the contrapositive of $p \rightarrow q$.
 (c) The inverse of the implication $p \rightarrow q$ is logically equivalent to $p \rightarrow q$.
 (d) The contrapositive of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.

6. Circle or otherwise indicate the rows of the truth table for which the following compound proposition is *false*:

$$(p \vee \neg q) \oplus (q \leftrightarrow r)$$

p	q	r	$p \vee \neg q$	$q \leftrightarrow r$	$(p \vee \neg q) \oplus (q \leftrightarrow r)$	
T	T	T	T	T	F	(1)
T	T	F	T	F	T	
T	F	T	T	F	T	
T	F	F	T	T	F	(4)
F	T	T	F	T	T	
F	T	F	F	F	F	(6)
F	F	T	T	F	T	
F	F	F	T	T	F	(8)

false for rows 1, 4, 6, 8

7. Use the truth table to prove the following logical equivalence:

$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

(This logical equivalence might be helpful in identifying an alternative strategy for a direct proof.)

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \rightarrow (p \vee q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

columns same, so
logically equivalent

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Questions 1-5 are 1pt each with no partial credit.

Questions 6-7 are 2pts each, and all work must be shown.

- b 1. Which of the following propositions is correct?
- (a) The inverse of the implication $p \rightarrow q$ is logically equivalent to $p \rightarrow q$.
 - ☒ (b) The converse of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
 - (c) The contrapositive of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
 - (d) The converse of the implication $p \rightarrow q$ is logically equivalent to the contrapositive of $p \rightarrow q$.
- b 2. If x , y , and z are real numbers, $x < y < z$ means that x is less than y and y is less than z . Which of the following is the negation of $x < y < z$?
- (a) $x \geq y \geq z$
 - ☒ (b) $x \geq y$ or $y \geq z$
 - (c) $x > z$
 - (d) $x > y > z$
- C 3. Which of the following is the negation of the statement "I drive to work if and only if it is rainy?"
- (a) If I drive to work, then it is not rainy.
 - (b) I drive to work if it is not rainy.
 - ☒ (c) I drive to work if and only if it is not rainy.
 - (d) I do not drive to work if and only if it is not rainy.
 - (e) I do not drive to work if it is not rainy.
- a 4. Suppose r and w are the propositions
- r : "it is rainy"
 w : "it is windy"
- Express in English a compound proposition that is logically equivalent to $w \rightarrow \neg r$.
- ☒ (a) If it is rainy, then it is not windy.
 - (b) It is windy, but not rainy.
 - (c) Whenever it is not windy, it is rainy.
 - (d) Being windy is sufficient for it to be rainy.

C 5. Suppose m and a represent the propositions:

m : "you are a member of the committee"

a : "you attend the meeting."

Express in symbols the compound proposition

"To be a member of the committee it is necessary that you attend the meeting."

(a) $\neg m \rightarrow a$

(b) $a \rightarrow \neg m$

(c) $m \rightarrow a$

(d) $a \rightarrow m$

6. Circle or otherwise indicate the rows of the truth table for which the following compound proposition is *true*:

$$(p \leftrightarrow \neg q) \oplus (q \wedge r)$$

p	q	r	$p \leftrightarrow \neg q$	$q \wedge r$	$(p \leftrightarrow \neg q) \oplus (q \wedge r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	T	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	F	F	F

True for
Rows
1, 3, 4, 6

7. Use the truth table to prove the following logical equivalence:

$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

(This logical equivalence might be helpful in identifying an alternative strategy for a direct proof.)

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \vee (p \rightarrow r)$	$q \vee r$	$p \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

columns equal,
 $\therefore$ logically equivalent