Group Members: ____

Theorem 2.1 Uniqueness of the Identity. In a group G there is only one identity element.

Theorem 2.2 Cancellation. In a group G the right and left cancellation laws hold; that is, $ba = ca$ implies $b = c$; and $ab = ac$ implies $b = c$.

Theorem 2.3 Uniqueness of Inverses. For each element a in a group G, there is a unique element b in G such that $ab = ba = e$.

(1) Why does the cancellation law of groups imply that in every row or column of a Cayley table, each group element will appear exactly once? Give a clear argument or proof.

(2) Matrix inverses are a little different for entries mod n. Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, \mathbb{Z}_{11})$.

(Continued on reverse)

(3) Let G be a group, and suppose that a and b are any elements of G. Show that if $(ab)^2 = a^2b^2$, then $ba = ab$.

(4) (Carefully prove the following by induction. This means to specify the base case and the assumptions and conclusions of the inductive step.) Let G be a group, and suppose that a and b are any elements of G. Show that $(aba^{-1})^n = ab^na^{-1}$, for any positive integer n.

(5) (1 point extra credit if done by 9/9) In the definition of a group on p.43, replace condition 2 with the condition that there exists e in G such that $e \cdot a = a$ for all a in G, and replace condition 3 with the condition that for each a in G there exists a' in G with $a' \cdot a = e$. Prove that these weaker conditions (given only on the left) still imply that G is a group.