Group Members: ____

Definition. The *order* of a group G, denoted by $|G|$, is the number of elements in G. Either $|G| = \infty$, or $|G|$ is a positive integer; a finite group is sometimes indicated by $|G| < \infty$.

Definition. The *order* of an element g of a group G, denoted $|g|$, is the smallest positive integer n such that $g^n = e$ ($n \cdot g = 0$ in additive notation). If no such integer exists, we say g has *infinite order* and write $|g| = \infty$.

(1) Classify the elements of the groups $U(8)$ and $U(10)$ according to their orders (see Group Activity 2A Problem (6)).

(2) Classify the elements of the group $\mathbb{R}^*$ under multiplication according to their orders.

(3) Classify the elements of the group $\mathbb{Z}_{12}$ under addition mod n according to their orders.

(4) Classify the elements of the group $\mathbb{Z}$ under addition according to their orders.

Definition. If a subset H of a group G is itself a group under the operation of G, we say that H is a subgroup of G.
If H is a subset of some G that we already know is a group, we have a head start on proving that H itself is a group.

What we know already: (i) the candidate binary operation on H is the one on G, (ii) the candidate binary operation on H is associative by inheritance.

What we must show about H: (i) the candidate binary operation is **closed** on H, (ii) H has an identity (the identity of G), and (iii) H contains inverses of all of its elements.

(Continued on reverse)

(5) List 5 subgroups of the nonzero complex numbers $\mathbb{C}^*$ under multiplication.

(6) By inspecting the Cayley tables of $U(8)$ and $U(10)$, list all of the subgroups of $U(8)$ and $U(10)$.

$U(8)$: $U(10)$:

(7) By inspecting the Cayley table of D_3, list all of its subgroups. Visualize the result of restricting to certain rows and columns. (Hint: there are 6 subgroups.)

D_3	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_0	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_{120}	R_{120}	R_{240}	R_0	F_3	F_1	F_2
R_{240}	R_{240}	R_0	R_{120}	F_2	F_3	F_1
F_1	F_1	F_2	F_3	R_0	R_{120}	R_{240}
F_2	F_2	F_3	F_1	R_{240}	R_0	R_{120}
F_3	F_3	F_1	F_2	R_{120}	R_{240}	R_0

Break. Theorem 3.1 One-Step Subgroup Test. Let G be a group and H a nonempty subset of G. If ab^{-1} is in H whenever a and b are in H, then H is a subgroup of G. (In additive notation, if $a - b$ is in H whenever a and b are in H, then H is a subgroup of G.)

Usage. 1. Identify the defining condition for H. 2. Prove the identity e of G fulfills this condition. 3. Assume some a, b in G fulfill the condition. 4. Prove that for this a, b that ab^{-1} fulfills the condition.

(8) Use the One-step subgroup test to prove that the even integers are a subgroup of $\mathbb{Z}$ under addition.

(9) Use the One-step subgroup test to prove that the subset H of an Abelian group G defined by

$$H = \{g \in G \ : \ |g| \leq 2\},$$

that is, the subset of elements with order at most 2, is a subgroup of G.