Group Members: ____

Theorem 6.3 Properties of Isomorphism Acting on Groups.
Suppose that ϕ is an isomorphism from a group G onto a group $\overline{G}$. Then
1. G is Abelian iff $\overline{G}$ is Abelian.
2. G is cyclic iff $\overline{G}$ is cyclic.
3. ϕ^{-1} is an isomorphism from $\overline{G}$ onto G.
4. If K is a subgroup of G, then $\phi(K) = \{\phi(k) \mid k \in K\}$ is a subgroup of $\overline{G}$.

(1) Describe how to apply Theorem 6.2 to prove Theorem 6.3 part 1. Technical details are not necessary.

(2) Describe how to apply Theorem 6.2 to prove Theorem 6.3 part 2. Technical details are not necessary.

(3) We know that if ϕ is a bijection, then so is ϕ^{-1}. Prove that ϕ^{-1} is operation preserving in order to prove Theorem 6.3 part 4.

(4) Use a subgroup test to prove Theorem 6.3 part 4. This should look similar to a problem on Exam 1.

Definition: Automorphism.
An isomorphism from a group G to itself is called an *automorphism* of G.

Definition: Inner Automorphism Induced by $a \in G$.
Let G be a group, and let $a \in G$. The function ϕ_a defined by $\phi_a(x) = axa^{-1}$ for all x in G is called the inner automorphism of G induced by a.

(5) Recall that Theorem 6.2 part 5 says that an isomorphism preserves the order of an element. That means for an automorphism ϕ of a cyclic group $\mathbb{Z}_n$, $\phi(1)$ must be a generator of $\mathbb{Z}_n$. Fill out the table with the automorphisms of $\mathbb{Z}_8$ by specifying for each automorphism where each element is mapped.

$\mathbb{Z}_8$	1	2	3	4	5	6	7	0
ϕ_a								
ϕ_b								
ϕ_c								
ϕ_d								
ϕ_e								
ϕ_e								

(6) Now compute the compositions $\phi_b \circ \phi_c$ and $\phi_c \circ \phi_d$. What are those results?

$\mathbb{Z}_8$	1	2	3	4	5	6	7	0	Result
$\phi_b \circ \phi(c)$									
$\phi_b \circ \phi(d)$									

(7) Prove that if G is Abelian, then there is a unique inner automorphism of G.

(8) Refer to Table 5.1 on page 105 for this question. Compute the inner automorphisms ϕ_{α_1} and ϕ_{α_5} by tabulating the image of each permutation in A_4 under ϕ_{α_5}. Do by looking up entries in Table 5.1 and without actually computing products of permutations.

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}
ϕ_{α_1}											
ϕ_{α_5}											

For thought. Visualize the inner automorphism ϕ_{α_5} by practicing the rotations on the tetrahedron.