Group Members:

Definition: External Direct Product

Let G_1, G_2, \ldots, G_n be a finite collection of groups. The *external direct product* of G_1, G_2, \ldots, G_n , written as $G_1 \oplus G_2 \oplus \cdots \oplus G_n$, is the set of all *n*-tuples for which the *i*th component is an element of G_i , and the operation is componentwise:

 $(g_1, g_2, \dots, g_n)(g'_1, g'_2, \dots, g'_n) = (g_1g'_1, g_2g'_2, \dots, g_ng'_n)$

(notice the left-right order is preserved). Component-wise group operation means that in the *i*th entry, $g_i g'_i$ is computed in the group G_i .

(1) Write down the elements of $\mathbb{Z}_3 \oplus \mathbb{Z}_5$.

(2) Verify computationally that $\mathbb{Z}_3 \oplus \mathbb{Z}_5 = \langle (1,1) \rangle$.

Theorem 8.1: Order of an element in a direct product

The order of an element in a direct product of a finite number of finite groups is the least common multiple of the orders of the components of the element. In symbols,

 $|(g_1,\ldots,g_n)| = \operatorname{lcm}(|g_1|,\ldots,|g_n|).$

Corollary of Theorem 4.4: Number of Elements of Order d in a Finite Group

In a finite group, the number of elements of order d is divisible by $\phi(d)$. (This is because the order d elements come from distinct cyclic subgroups of order d – these subgroups may overlap, but the overlap does not contain any order d elements. As a result you can compute the number of cyclic subgroups of order d by knowing the number of order d elements.)

(3) Determine the number of cyclic subgroups of order 15 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$.