PRINT Last name:________________ First name:________________

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Math 454 Exam 1, Fall 2005

I. Examples and Counterexamples. Do not give proofs, but clearly indicate your proposed example or counterexample.

1. Draw a graph which contains a *maximal* independent set which is not a *maximum* independent set. Identify these two independent sets.

2. Give an example of a closed trail which is not a cycle.

3. Give a graph G and express it as the *graph union* of a list of its subgraphs, but where that list is not a *graph decomposition* of G.

4. Give a single list containing at least 2 positive numbers which is the degree sequence of a graph but not of a simple graph. Draw the graph.

5. Draw an example of a weakly connected directed graph with more than one strongly connected component. Circle the strong components and identify all strong components with no edges leaving them (sink components).

6. Give an example of a graph with chromatic number $\chi(G) = 4$ but which contains no K_4. (Hints: $\chi(G) = 4$ means 4-partite but not 3-partite. There is one such graph of order 6.)

II. Constructions and Algorithms. Do not write proofs, but do give clear, concise answers.

7. **(a)** Give a drawing of the simple graph G with $V(G) = \{t_1t_2 : t_1, t_2 \in \{0, 1, 2\}\}$ (the set of ternary 2-tuples) and edges $xy \in E(G)$ iff x and y differ in exactly one position. (Hint: $|V(G)| = 9$.)

   (b) Determine $e(G)$, the number of edges of G.

   (c) Give a closed trail containing all vertices of G. Present the trail by listing its vertices in order.

   (d) Find a bipartite subgraph of G with at least $e(G)/2$ edges.

III. Proofs.

8. Let $k \geq 1$. Assume G is a simple graph with chromatic number $\chi(G) = k$. Prove that the complement $\overline{G}$ of G has a clique of size at least $n(G)/k$.

9. For this problem, you may use Theorem 1.2.26:

   > A graph G is Eulerian iff it has at most one nontrivial component and all its vertices are even.

   Now prove that if G is a graph with c components and exactly $2k$ odd vertices, where $c, k \geq 0$, then G has a decomposition into $c + k$ or fewer trails.